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THE SYNTHESIS OF SEQUENTIAL SWITCHING CIRCUITS *

BY

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ABSTRACT

An orderly procedure is developed by which the requirements of a sequential switching circuit (one with memory) can be reduced to the requirements of several combinational switching circuits (those without memory). Important in this procedure are:

1. the flow table: a tabular means by which the requirements of a sequential switching circuit may be stated precisely and by which redundancy in these requirements may be recognized and eliminated, and
2. the transition index: a new variable which indicates the stability (or lack of stability) of a switching device.

The role of those switching devices which are not directly controlled by the input of a sequential switching circuit is investigated thoroughly. The resulting philosophy, which is exploited in synthesis procedures for circuits using either relay or vacuum-tube switching devices, is valid for circuits using other devices as well.

PART I †

I. INTRODUCTION TO RELAY CIRCUIT THEORY

Historical Background

In 1938, C. E. Shannon established an orderly algebraic procedure for the treatment of relay contact networks.[2] This major theoretical advance was based on an analogy with the calculus of propositions used in symbolic logic. In 1948, G. A. Montgomerie [3] described a concise

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† Part II will be published in this JOURNAL for April, 1954.

[2] C. E. Shannon, "A Symbolic Analysis of Relay and Switching Circuits," *Trans. AIEE*, Vol. 57, pp. 713–723 (1938).

[3] G. A. Montgomerie, "Sketch for an Algebra of Relay and Contactor Circuits," *Jour. IEE*, Vol. 95, Part III, pp. 303–312 (1948).

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method of specifying static (or "combinational") relay circuits which has since been used extensively by Shannon [4] and others, and is now called a "table of combinations." The need for similar orderly analysis methods for circuits with dynamic relay action was realized by Montgomerie,[5] and some of the matrices described here bear superficial resemblance to his, even though they were developed independently.

Assumed Form of the Switching Circuit

Our generalized relay switching circuit will have p input terminals, and q output terminals. Each of the input terminals will be connected to a terminal (called the *control terminal*) of the coil winding of a single-winding relay; the other end of the winding will be connected to the proper supply voltage. (See Fig. 1, for example.) These relays, which are under the direct control of the circuit inputs, are designated as X or *primary* relays. All other relays (if any) in the circuit are to be called Y or *secondary* relays. Their number will be designated s.

Each of the s secondary relays will be controlled by a network of contacts from (in general) all of the relays—both primary and secondary—in the circuit. Similar networks of contacts (the Z networks) will lead to each of the q output terminals from ground.

Contacts associated with a given relay (R) will be given the lower-case designations. *Normally open* contacts are labeled with the *unprimed* letter (r), and the *normally closed* contacts with the *primed* letter (r'). We shall assume that, for a given relay:

1. All normally open contacts are open (or closed) at the same time. Likewise, all normally closed contacts are open (or closed) simultaneously.
2. When the normally open contacts for a given relay are open (or closed), the normally closed contacts are closed (or open), and vice versa.

Algebraic Description of the Relays

A *binary* (two-valued) variable is one which may, at any given time, assume just one of two possible *complementary* values. The binary variables used in describing a relay circuit represent the absence or presence of a metallic path. These variables are assigned the values zero and one. A contact, or a network of contacts, has a *transmission of unity* when it is *closed*, and a *zero transmission* when it is open.[6]

In terms of the transmission concept, $r = 0$ will be interpreted to mean: The normally open contacts on the relay R are open; that is, the

[4] C. E. Shannon, "The Synthesis of Two-Terminal Switching Circuits," *Bell System Tech. Jour.*, Vol. 28, pp. 59–98 (1949).

[5] Transmission is thus somewhat analogous to admittance. The dual concept, that of *hindrance*, is somewhat analogous to impedance; a closed circuit has a zero hindrance and an open circuit has unity hindrance. We shall use the idea of transmission exclusively.

relay is *unoperated*. Similarly $r = 1$ will mean: The normally open contacts on R are closed; that is, the relay is *operated*. Of course—with the assumptions made previously—whenever $r = 0$, then $r' = 1$ and whenever $r = 1$, then $r' = 0$. Thus r and r', when considered as contact variables, always have complementary values.[6]

A relay is *energized* or *de-energized* according as the transmission of the contact network in series with its winding has a transmission of unity or zero, respectively. It will be convenient to use the upper-case letter R as the notation for the transmission of this network. Thus, when $R = 1$, the relay is energized; and when $R = 0$, the relay is de-energized.

It is necessary to make a fine distinction between the state of operation of a relay and its state of energization, as reflected by the variables r and R, respectively. If r and R have the same value the relay is in a *stable* state. If instead, r and R have complementary values the relay is in an unstable state. Since both r and R may each have only the values zero and one, there are just four mutually exclusive situations which may arise:

1. $r = 0$ and $R = 0$; a stable state.
2. $r = 0$ and $R = 1$; an unstable state; the relay is unoperated and is energized. If R continues to have the value one, then eventually the relay will become operated and the value of r will change from zero to one. The time required is called the relay *operate* time.
3. $r = 1$ and $R = 0$; an unstable state; the relay is operated and is de-energized. If R continues to have the value zero, then eventually the relay will become unoperated and the value of r will change from one to zero. The time required is called the relay *release* time.
4. $r = 1$ and $R = 1$; a stable state.

We now define a new variable, τ_R, called the *transition index* which reflects the stability or instability of a relay R. When $\tau_R = 0$ the relay is stable; when $\tau_R = 1$ the relay is unstable.

For convenience we will make use of an algebraic operation which we shall call cyclic addition,[7] and for which the notation is $\oplus$. It will have the following properties:

$$0 \oplus 0 = 1 \oplus 1 = 0; \qquad 0 \oplus 1 = 1 \oplus 0 = 1. \tag{1}$$

In terms of this notation

$$\tau_R = r \oplus R \qquad \text{and} \qquad R = r \oplus \tau_R. \tag{2-a, b}$$

[6] The priming notation may be used whenever the idea of complementation exists. Thus, $0' = 1$ and $1' = 0$.

[7] In the language of congruences, τ_R is congruent, modulo two, to the sum of r and R. See, for example, Birkoff and MacLane, "A Survey of Modern Algebra," New York, The Macmillan Company, 1941, pp. 23–29.

Combinational and Sequential Circuits

Consider for the time being a switching circuit which has no secondary relays; for example, that of Fig. 1(*a*). For this kind of circuit the contact networks which lead to the output terminals must be composed of contacts from relays which are under the direct influence of the input (the primary relays). Since each primary relay may be either operated or unoperated, there are 2^p possible states of operation for the p primary relays, taken collectively. Each of these *primary states* determines uniquely a transmission of either zero or unity at each output terminal. In Fig. 1(*a*), for instance, there is an output ground if X_1 is operated or if X_2 is unoperated, or both. If X_1 is unoperated and if X_2 is operated, however, the output will be ungrounded.

Consider now a circuit having secondary relays as well as primary relays—for example that of Fig. 1(*b*). Assume that all three of the relays are unoperated. Operation of the X_2 relay has no effect on the Y relay but does result in a ground at the output terminal (since the y' contact is closed). As long as Y remains unoperated, the output is grounded or not grounded according to whether X_2 is or is not operated.

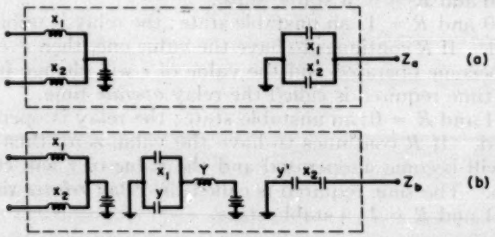


FIG. 1. Illustrating combinational and sequential switching circuits.

Let us now assume that the X_1 relay is operated. The closing of its normally open contact, x_1, energizes and subsequently operates the Y relay, and closes the y contact. Now the state of operation of X_1 cannot affect Y since the latter is permanently energized through its own normally open contact. (Permanently, that is, until the supply voltages are removed.) Now the y' contact in the output network is open and operation of X_2 cannot ground the output terminal.

Notice that the output transmission is not a unique function of the primary relay states of operation but depends as well on the past history of the circuit. The circuit may be said to have a *memory;* it "remembers" whether or not X_1 has been operated.

We may generalize from these two simple examples: In a circuit having no secondary relays there can be no "memory"; the states of operation of the primary relays uniquely determine the output trans-

missions. Such a circuit is called a *combinational* circuit. In a circuit having secondary relays, the possibility of a "memory" exists since the states of operation may not uniquely determine the output transmissions. A circuit having secondary relays will be called a *sequential* circuit.

Specification of the Terminal Characteristics of Switching Circuits

The terminal characteristics of a combinational circuit may be described equally well by specifying the nature of the contact networks at the outputs. A useful means of specification of these networks (or of *any* two-terminal contact network) is the *table of combinations*. In it are listed the 2^p possible (collective) states of operation of the p primary relays which may contribute contacts to such an output network; beside each of these 2^p states is listed the transmission of the network. In Table I, the table of combinations tells us that the Z_a network (Fig. 1(*a*)) has a transmission of zero if and only if X_1 is unoperated and X_2 is operated.

TABLE I.—*Tables of Combinations for the Networks in Fig. 1.*

(*a*)

x_1	x_2	Z_a
0	0	1
0	1	0
1	0	1
1	1	1

(*b*)

x_1	y	Y
0	0	0
0	1	1
1	0	1
1	1	1

(*c*)

x_2	y	Z_b
0	0	0
0	1	0
1	0	1
1	1	0

The precise specification of a sequential circuit is more difficult. We can list the transmissions of the Y and Z networks in tables of combinations (see Table I(*b*) and (*c*)) but this listing tells us little about the terminal characteristics of the circuit or what function "memory" plays in its operation.

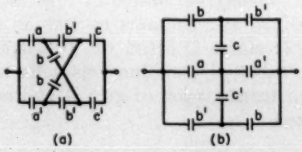
Up to the present time no precise way of tabulating the terminal characteristics of the general sequential switching circuit has been published. The *flow table* developed in Section II is the author's answer to this problem. Without an exact specification, such as the one the flow table gives, we cannot hope to be able to lay down rules for the synthesis of the general sequential circuit.

Comments on the Synthesis of Contact Networks

Our synthesis method for sequential circuits will lead to the listing of the transmission requirements for several Y and Z networks in tables of combinations. The physical realization of contact networks which meet the requirements in a table of combinations has received much study and there is much research yet to be done. We will not attempt

to show how to design such networks but will usually diagram one possible physical realization and the reader may verify that it corresponds to its table of combinations.[8]

Part of the problem of synthesizing contact networks exists because several quite different-appearing networks may be terminally equivalent. It is not always easy to decide on a criterion of merit for these different networks or to be sure that once a "good" network is found a "better" one does not also exist. However, a common denominator for all equivalent contact networks is the table of combinations. For example, Fig. 2 gives two equivalent networks along with the table of combinations which they have in common.



a	b	c	Network Transmission
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

(c)

FIG. 2. Two equivalent networks and their table of combinations.

II. THE ANALYSIS OF SEQUENTIAL RELAY CIRCUITS

Matrix Representation for Two-Terminal Networks

In each of the relay switching circuits discussed in this paper, there is a two-terminal contact network connected between ground and the control terminal of each secondary relay. These networks have been given the notations $Y_1, Y_2, \cdots, Y_s$. In addition, there are two-terminal networks between ground and each of the output terminals. These networks are to be given the notations $Z_1, Z_2, \cdots, Z_q$. Each of the Y and Z networks may be a function of all the contact variables for the circuit—that is, of the $(p + s)$ variables $x_1, x_2, \cdots, x_p, y_1, y_2, \cdots, y_s$. Therefore, each of these transmission functions may be represented in a table of combinations with rows corresponding to all possible combinations of the $(p + s)$ contact variables. It is more convenient, however, to use modified tables of combinations which, for the sake of compactness, are put in the form of rectangular matrices.

To illustrate the construction of such matrices, Table II shows the tables of combinations for the three secondary relay control networks

[8] For further study on contact networks the reader is referred to W. Keister, A. E. Ritchie and S. Washburn, "The Design of Switching Circuits," New York, D. Van Nostrand Company, Inc., 1951.

For recent papers on the reduction of combinational circuit requirements to simple forms, refer to M. Karnaugh, "The Map Method for Synthesis of Combinational Logic Circuits," AIEE Technical Paper No. 53-217, April, 1953; or to E. E. Veitch, "A Chart Method for Simplifying Truth Functions," *Proc. Assn. for Computing Machinery*, May, 1952; or to W. H. Burkhart, "Theorem Minimization," *ibid.*

TABLE II.—*Tables of Combinations for the Y and Z Networks of Fig. 3.*

x_1	x_2	y_1	y_2	y_3	Y_1	Y_2	Y_3	Z
0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	1	0
0	0	0	1	0	1	1	0	0
0	0	0	1	1	0	1	1	0
0	0	1	0	0	0	0	0	0
0	0	1	0	1	1	0	1	1
0	0	1	1	0	1	1	0	0
0	0	1	1	1	0	1	1	0
0	1	0	0	0	1	0	0	0
0	1	0	0	1	1	0	0	0
0	1	0	1	0	0	0	0	0
0	1	0	1	1	0	0	0	0
0	1	1	0	0	1	0	0	0
0	1	1	0	1	1	0	0	0
0	1	1	1	0	1	1	1	0
0	1	1	1	1	1	1	1	0
1	0	0	0	0	0	1	0	0
1	0	0	0	1	0	0	1	0
1	0	0	1	0	0	1	0	0
1	0	0	1	1	0	0	1	0
1	0	1	0	0	1	0	0	0
1	0	1	0	1	1	0	0	0
1	0	1	1	0	1	0	0	0
1	0	1	1	1	1	0	0	0
1	1	0	0	0	1	0	0	0
1	1	0	0	1	1	0	0	0
1	1	0	1	0	0	0	0	0
1	1	0	1	1	0	0	0	0
1	1	1	0	0	1	0	0	0
1	1	1	0	1	1	0	0	0
1	1	1	1	0	1	0	0	0
1	1	1	1	1	1	0	0	0

and the single output network of the circuit of Fig. 3. Table III gives the corresponding matrix forms. In general, the Y matrix will have as entries ordered s-tuples of the proper values of $Y_1, Y_2, \cdots, Y_s$; and similarly, the Z matrix will have as its entries ordered q-tuples of the proper values of $Z_1, Z_2, \cdots, Z_q$. Positions in the matrix will be given in the form $(x;y)$, where x is the ordered p-tuple of the values of the primary contact variables $x_1, x_2, \cdots, x_p$, and where y is the ordered s-tuple of the values of the secondary contact variables $y_1, y_2, \cdots, y_s$. For example, the entry "1" in the (00;101) position of the Z matrix of

Table III(*b*) corresponds to the starred row of Table II. In this same starred row the values for Y_1, Y_2, and Y_3 are one, zero, and one, respectively. And so the entry in the (00;101) position of the Y matrix of Table III(*a*) is "101."

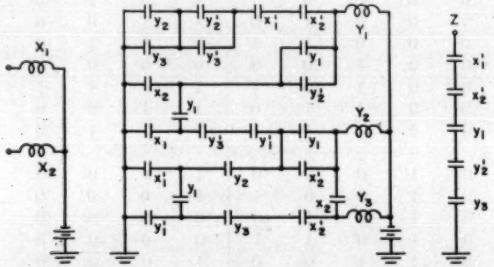


FIG. 3. A sequential switching circuit.

TABLE III.—*Y and Z Matrices Formed from the Data of Table II.*

(*a*)
The *Y* Matrix

x: / *y*:	00	01	10	11
000	000	100	010	100
001	101	100	001	100
010	110	000	010	000
011	011	000	001	000
100	000	100	100	100
101	101	100	100	100
110	110	111	100	100
111	011	111	100	100

(*b*)
The *Z* Matrix

x: / *y*:	00	01	10	11
000	0	0	0	0
001	0	0	0	0
010	0	0	0	0
011	0	0	0	0
100	0	0	0	0
101	1	0	0	0
110	0	0	0	0
111	0	0	0	0

The Composite Transition Matrix

Next we shall form the *composite transition matrix*, τ, for our present example. We shall make use of the equations

$$\tau_{Y1} = Y_1 \oplus y_1, \quad \tau_{Y2} = Y_2 \oplus y_2, \quad \text{and} \quad \tau_{Y3} = Y_3 \oplus y_3, \quad (3\text{-}a, b, c)$$

which correspond to Eq. 2(*a*) applied to the three secondary relays. The mechanics of construction of τ are as follows:

For a given primary relay state, $x = (x_1, x_2, \cdots, x_p)$, and for a given secondary relay state $y = (y_1, y_2, \cdots, y_s)$—that is, for a given *total* relay state $(x;y)$,—there is an entry $(Y_1, Y_2, \cdots, Y_s)$ in the Y matrix. In τ, the proper entry at the $(x;y)$ position is $(\tau_{Y1}, \tau_{Y2}, \cdots, \tau_{Ys})$

$= (Y_1 \oplus y_1, Y_2 \oplus y_2, \cdots, Y_s \oplus y_s)$. For instance, the entry in the (00;010) position of the Y matrix of Table III(a) is "110." Since this entry is in the row corresponding to $y = 010$, then we find the proper entry in the τ matrix of Table IV(a) by adding cyclicly the corresponding components of "110" and "010." The result is "100," and this is inserted in the (00;010) position of the τ matrix. This derived entry is to be interpreted: "For the total relay state (00;010), the relay Y_1 is in an unstable state, but the relays Y_2 and Y_3 are both in stable states."

TABLE IV.—*The τ Matrix Derived from Table III.*

y: \ x:	00	01	10	11
000	000	100	010	100
001	100	101	000	101
010	100	010	000	010
011	000	011	010	011
100	100	000	000	000
101	000	001	001	001
110	000	001	010	010
111	100	000	011	011

Later, in the synthesis process, we shall need to reverse the procedure above and derive the Y matrix from the τ matrix. Then we shall need to make use of the equations

$$Y_1 = \tau_{Y1} \oplus y_1, \qquad Y_2 = \tau_{Y2} \oplus y_2, \cdots, Y_s = \tau_{Ys} \oplus y_s, \tag{4}$$

which correspond to Eq. 2(b) applied to the s secondary relays. In order to derive each entry in the Y matrix, we shall have to add cyclicly the components of the appropriate value of $y = (y_1, y_2, \cdots, y_s)$ to each component of the corresponding entry in τ.

In a τ matrix, each entry consisting entirely of zeros indicates stability for all the secondary relays. If an entry contains a single digit one as a component, then just one secondary relay is in an unstable state, and the entry tells us what the secondary relay state will next be if the input state remains unchanged. For instance, in the τ matrix of our present example, the "100" entry at the (00;010) position indicates that the Y_1 relay is unstable; and thus, if the input state remains "00," the resulting total relay state will be (00;110). A glance at τ reveals that all the secondary relays are stable for this new total relay state, since the entry at this position consists entirely of zeros. The circuit action described above is diagrammed by the heavy arrow in Fig. 4.

An important fact to observe in the use of a composite transition matrix is that a change of state in the secondary relays is indicated by

vertical movement in the matrix and in the associated diagram, while a change of input state is accompanied by motion in a horizontal direction. Moreover, once a stable state has been reached for all secondary relays, then further circuit changes can occur only if modification is made in the input state. In other words, if the circuit has a stable secondary relay state, and if the input state is changed from the existing value to another one, then the focal point of our attention must first be moved horizontally into the column corresponding to the new input state; and if further changes are to occur, these must be in the vertical direction.

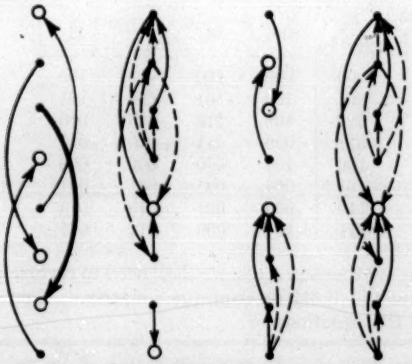


FIG. 4. Transition diagram for the matrix given in Table IV.

Race Conditions

In a composite transition matrix the presence of two or more of the digits "1" indicates that at least two secondary relays are simultaneously unstable, and that a *race condition* exists. In such a case, several secondary relay actions are possible, depending upon the magnitudes of the operating and release times for the relays involved and upon the past history of their excitations. For example, in Table IV, if the total relay state is (10;111), the corresponding entry is "011." This entry indicates that a "race" develops between the relays Y_2 and Y_3 to see which relay will become released first. If Y_2 wins the "race" and is released first, the resulting secondary relay state is "101." If, however, Y_3 is the first relay to release, then the secondary relay state becomes "110." But if a "tie" develops and both relays release at the same time, the secondary relay state will be "100." These three possibilities are indicated by dotted lines rising from the bottom node in the third column of Fig. 4.

All race conditions in Fig. 4 are indicated by dotted lines. In this figure each race condition is *non-critical*, since each of the alternate possibilities leads eventually to the same *ultimate* secondary relay state. For example (see the right-hand column of Fig. 4): If the input state is "11," the ultimate secondary relay state will be "100," no matter what the initial secondary relay state was.

Not all race conditions are of the noncritical variety. It may be that the alternate possibilities present in a race condition lead to different ultimate circuit conditions. Consider, for instance, Table V (and Fig. 5). Here, several ultimate circuit conditions are possible if the total relay state is initially (00;010).

TABLE V.—*A τ Matrix Illustrating Race Conditions and Cycles.*

ϕ y: \ x:	00	01	10	11
000	010	000	000	001
001	000	100	011	010
010	101	001	100	110
011	010	010	100	001
100	000	010	000	001
101	000	001	000	010
110	000	100	100	001
111	010	000	100	100

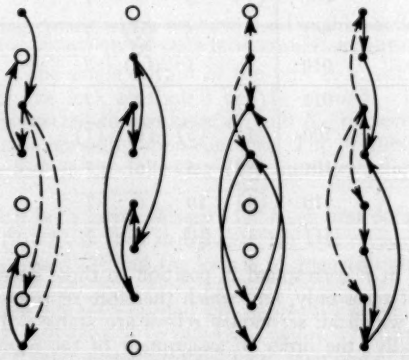


FIG. 5. Transition diagram for τ matrix of Table V.

Cycles and Ultimately Stable Terminal Action

It may be that, for a given input state, internal circuit action is continuously sustained because at least one secondary relay is always maintained in an unstable state. For this situation there will be no

single ultimate secondary relay state, but instead there will be a *cycle* of states occurring. The column of Table V (and of Fig. 5) for which the input state is "01" illustrates such a cycle. Here the secondary relay states $\cdots$, 001, 101, 100, 110, 010, 011, $\cdots$ occur in cyclic fashion.

If the output state of a relay switching circuit remains constant for all the secondary relay states of a cycle, we shall say that the circuit has *ultimately stable terminal action.*

It is also possible that race conditions and cycles may coexist in a switching circuit. Two examples of this situation are given in the columns of Table V which correspond to the input states "10" and "11."

The Flow Table

Our attention will now be limited to the analysis of those relay switching circuits in which there are no cycles, and where all race conditions (if any exist) are of the noncritical type. For this kind of circuit we can construct a *flow table, F.* Our running example (Fig. 3, Tables II, III, and IV, and Fig. 4) is of the proper classification, and the associated flow table is given in Table VI.

TABLE VI.—*The Flow Table Derived from Table IV and Fig. 4.*

y: \ x:	00	01	10	11
000	(1)	5	3	7
001	8	5	(2)	7
010	9	5	(3)	7
011	(4)	5	2	7
100	1	(5)	(6)	(7)
101	(8)	5	6	7
110	(9)	10	6	7
111	4	(10)	6	7

The circles in F correspond in position to those entries in τ which are made up of zeros only, and which therefore represent those circuit conditions for which all secondary relays are stable. The circles are numbered serially; the order of assignment of the numbers is unimportant. The remaining entries in F are uncircled, and each such entry tells what stable circuit condition will ultimately result if the circuit is put in a total relay state corresponding to the entry in question and if the input state remains unchanged. This stable circuit condition can be derived by analyzing the composite transition matrix, τ. With this rule, and aided by the diagram of Fig. 4, if necessary, we obtain Table VI.

Since we are going to state our synthesis problems in terms of flow

tables, it will be worth while to take time to understand thoroughly how a flow table is used and what its entries mean. The flow table can give us a graphic idea of what circuit actions can occur in a relay circuit. Just as in the composite transition matrix, horizontal motion in the flow table corresponds to modification of the input state. Within each row of the table the following rules hold:

1. Each circled entry in a row of a flow table indicates a stable circuit condition, and no further changes will occur unless the input state is modified.
2. Each circled circuit condition within a row of a flow table can lead to any other circuit condition (circled or uncircled) which is listed in the same row of the table.
3. Each uncircled entry in a row of a flow table indicates the stable circuit condition which will ultimately follow if the input state is left constant. This stable circuit condition, circled, will be listed in the same column of the table, but in another row.

For example (see the fifth row of Table VI): If the existing circuit condition is denoted by "6," then circuit conditions "1," "5," or "7" can follow, depending upon whether the input state is changed from "10" to "00," "01," or "11," respectively. If, particular, circuit condition "1" results (by changing the input state from "10" to "00"), we must next concentrate our attention on the first row of the flow table, because the circled "1" entry appears in this row.

The switching circuit which we have used in this section on analysis is one which is designed to be used in conjunction with an electrically operated lock. The single output of the unit is to act as the switch which energizes the lock so that it can be opened. The two input terminals are connected to two keys, K_1 and K_2, respectively—each of which has a single normally open contact. The "combination" which is to open the lock is the following (the numbers refer to entries in Table VI):

Starting with both keys released (1), K_1 is first depressed (3) and then released (9); K_2 is then depressed (10) and released (4); finally, K_1 is again depressed (2) and the lock is to open (8) when K_1 is then released.

After the lock has been opened (8), depressing either or both of the keys (5, 6, or 7) will allow it to be locked again. If a mistake is made in working the "combination" for the lock, then one of the circuit conditions 5, 6, or 7 will result, and the combination may be started again only after both keys are first released (1).

Summary of Material on Analysis

The transition index τ_R has been defined. Its use led to the composite transition matrix τ. τ and Z (the output matrix) give us all the

information that the original circuit diagram can. With a knowledge of the succession of input states, we may determine—with the aid of τ—the states of each relay of the switching circuit. Then, by looking at the proper entry in Z, the output state is determined.

We have concerned ourselves only with the relative order of state changes, and not with the actual timing in the circuit. With this outlook, we have seen some of the situations which may arise. For a circuit with no sustained cycles [9] or potentially critical race conditions, we have defined a flow table F which is useful in determining ultimate circuit conditions, if the input state is changed only after the secondary relays have reached stable states.

III. THE SYNTHESIS OF A SEQUENTIAL RELAY CIRCUIT

Introductory Remarks

In this section our attention will be limited to those design problems in which the input state changes only after a stationary output state has been reached, and in which such an output state will always occur: in other words, those circuit actions which could be those of a relay circuit with ultimately stable terminal action.

The most difficult and the most important part of a synthesis of a relay switching circuit—as with most other syntheses—is the problem of saying what we want to do. The flow table does for sequential relay circuits which have ultimately stable output what the table of combinations does for combinational-type contact networks. In each case the table gives in a precise way meanings that dozens of qualifying statements in a word specification might never be able to convey.

We must keep in mind the situation which faces the designer. First, he would like to specify what the circuit output state is to be for all possible sequences of input states, and not for just a single sequence. It is all very well to say what the output states should be for some "normal" sequence of inputs; but if there is even a possibility that other sequences might occur, then circuit action must be specified for these sequences also. A complete problem specification must indicate clearly what happens for each conceivable set of circumstances.

Secondly, if the circuit designer can honestly say that he cares only about the output state as some function of the input state, he cannot fairly say in the problem statement what secondary relays he wants to use. The problem statement can then be made only in terms of what signals are available (the input states) and what signals are desired (the output states).

[9] It is also possible to write flow tables for circuits with cycles or for circuits in which it is important to specify output states in addition to those which correspond to total relay states for which the circuit is stable. In such flow tables, entries of a type "ⓘj" may be utilized. An entry "ⓘj" is the "destination" of all entries with uncircled i's and it directs our attention to the entry with a circled "j" as the next circuit condition of interest.

The Problem Statement and the Flow Table

In order to illustrate the various problems which arise during the synthesis of a sequential circuit, we shall first limit ourselves to a specific example. The circuit we wish to design has some of the properties of a "delay line." It is to have two input terminals and two output terminals. We want the output state to be the same as the last previous input state. In order to illustrate how restrictions on the input state are taken care of, we shall specify that we shall use our circuit only under those circumstances which allow the input state to change by one variable at a time. In other words, for example, modification of the input state from "01" to "00" or to "11" will be allowed, but from "01" to "10" will be impossible. And so, after our synthesis is complete, it would not be fair to complain of improper circuit action for these forbidden changes of input state; for we specify here that they cannot occur.

With the restrictions named above, there will be just two separate circuit conditions possible for each input state, and each of these will be associated with a different output state. For instance, if the input state is "00," it could have been preceded by either "01" or "10." The corresponding output state then will be either "01" or "10," respectively.

TABLE VII.—*A Flow Table and Output Data for a "Delay Line" Circuit.*

(*a*) Flow Table				(*b*) Output Data	
x:				Stable Circuit Condition	Output State
00	01	10	11		
(1)	3	5		1	01
(2)	3	5		2	10
1	(3)		7	3	00
1	(4)		7	4	11
2		(5)	8	5	00
2		(6)	8	6	11
	4	6	(7)	7	01
	4	6	(8)	8	10

Consequently, there will be eight stable circuit conditions—two for each of the four possible input states. These are numbered serially and listed as circled entries in Column (*a*) of Table VII, each in a separate row and each in the proper column. The output state associated with each is listed in Column (*b*).

Next we list the uncircled entries of the flow table, making use of the three rules stated previously (see p. 173), and being careful to make certain the conditions in the word statement of the problem are satisfied.

The positions in the flow table which correspond to unallowable input transitions are left blank.[10]

The first row of our derived flow table informs us, for instance, that circuit condition "1" (which may occur for input state "00") may lead to either circuit condition "3" or "5," and Table VII, Column (*b*) verifies that each of these latter conditions will give the proper output state, "00."

Condensation of the Flow Table by Row Merging

From our discussion of flow tables in the section on analysis, we know that there is going to be one secondary relay state assigned to each row of the flow table. If our object is to reduce the number of secondary relays as far as possible, we would like to be sure that our

TABLE VIII.—*Hypothetical Derivation of a τ Matrix Corresponding to Table VII.*

(*a*) Flow Table

y: \ *x*:	00	01	10	11
00	(1)	(3)	5	7
11	(2)	3	(5)	8
10	1	(4)	6	(7)
01	2	4	(6)	(8)

(*b*) The τ Matrix

y: \ *x*:	00	01	10	11
00	00	00	11	10
11	00	11	00	10
10	10	00	11	00
01	10	11	00	00

problem has been stated in a flow table with as small a number of rows as possible. Sometimes it is possible to *merge* two rows of a flow table. Each such *merger* will reduce the number of rows by one. The rule for merging of rows in a flow table is this:

Two or more rows of a flow table may be merged if—and only if—for each input state, these rows do not have conflicting entries. An entry which appears in any one of the merged rows will appear in the composite row. Those entries which are circled in any one of the merged rows will be circled in the row resulting from the merger.

Application of this rule to Column (*a*) of Table VII indicates that the following pairs of rows may be merged: one and three, two and five, four and seven, and six and eight. The condensed flow table which results from making all four mergers is given in Table VIII.[11] The

[10] We may also leave a position in a flow table vacant if, for the corresponding input-state transition, we do not care to define the circuit action to follow. See the further discussion of this point in *Input-Output Sets*, page 183 of this paper.

[11] In our example the result of making all possible row mergers gives an answer which is independent of the order in which the mergers are made. In *Requirements for a Unique Flow-Table Condensation*, page 188 of this paper, however, it will be proved that the result of condensing a flow table to the point where no further mergers are possible may not be unique if (as in our example) there are "vacancies" in the original table: that is, if some modifications of input state are prohibited.

merger of rows one and three, for example, results in the first row of this new table.

The first row of Column (*a*) in Table VIII preserves the information contained in each of the component rows. It retains the information that circuit condition "1" leads to condition "3" or condition "5," depending on whether the input state is changed from "00" to "01" or to "10," respectively. Similarly, this first row tells us (as did the third row of Table VII, Column (*a*)) that circuit condition "3" may lead either to condition "1" or to condition "7," depending on the change of input state.

In the circuit which we shall synthesize from the condensed table, circuit condition "3" would lead to circuit condition "5" *if* the input state could be changed from "01" to "10." But the "if" is an enormous one; for we stated firmly during the specification of the problem that this transition in input state could not occur. Thus it is important to recognize that, since this information which the top row of Table VIII, Column (*a*) gives is about a hypothetical change in input state, it will neither be of value to us nor cause us trouble. Similar situations are apparent in each of the four rows in this same Column (*a*).

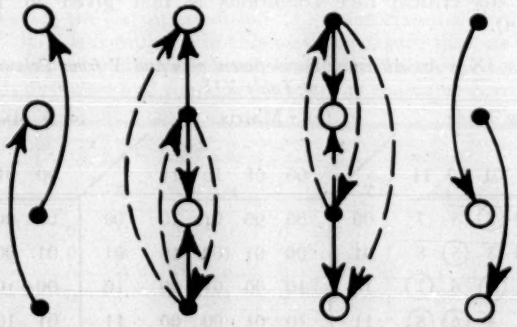


FIG. 6. Transition diagram for the derivation in Table VIII.

The Assigning of Secondary Relay States to the Rows of the Flow Table

Since there are four rows in the condensed flow table, it will be necessary to have at least two secondary relays in order to assign a distinct secondary relay state to each row. If we were to hypothesize that the operating and release times of each secondary relay were precisely the same, and that this time was the same for each of the secondary relays, then the problem of assigning secondary relay states to the rows of the flow table would be easy indeed. For then we could assign them in some arbitrary fashion to the rows, as has been done in Column (*a*) of Table VIII. The assumption that all operating and

release times were all the same would guarantee that all race conditions would end in "ties," and we could then form the composite transition matrix of Column (*b*) of Table VIII, which corresponds to the diagram of Fig. 6. The transitions which correspond to the "tied races" are indicated by the solid lines in the second and third columns of the figure.

From a practical point of view, the presence of race conditions such as those discussed above is perilous; for if the operating characteristics of a relay change slightly over a period of time, one or more of the transitions indicated by dotted lines becomes possible. These transitions would result in improper operation of the switching circuit.

We can avoid the difficulties of race conditions in which "ties" *must* be achieved in order for proper circuit operation to occur (and therefore in which some entries in τ *must* indicate two or more relays to be unstable). What we need to do is to assign secondary relay states to the flow table in such a way that in τ, only *one* secondary relay is indicated to be unstable at a given time. After having done this, we may then reconsider some of the entries in τ and allow, if we desire, *noncritical* race conditions to exist.

For our present example, one assignment of secondary relay states which avoids critical race conditions is that given in Table IX, Column (*a*).

TABLE IX.—*An Alternate Development of τ and Y from Column (a) of Table VII.*

(*a*) Flow Table

y: \ x:	00	01	10	11
00	(1)	(3)	5	7
01	(2)	3	(5)	8
10	1	(4)	6	(7)
11	2	4	(6)	(8)

(*b*) τ Matrix

y: \ x:	00	01	10	11
00	00	00	01	10
01	00	01	00	10
10	10	00	01	00
11	10	01	00	00

(*c*) Y Matrix

y: \ x:	00	01	10	11
00	00	00	01	10
01	01	00	01	11
10	00	10	11	10
11	01	10	11	11

Derivation of the τ and Y Matrices

The τ matrix corresponding to our choice of secondary relay states is given in Table IX, Column (*b*). By reversing the procedure of analysis, we add cyclicly to each entry in τ the secondary relay state assigned to the row in which the entry is found. The Y matrix of Column (*c*) results.

Derivation of the Z Matrix

By consulting the output data of Table VII, Column (*b*), we may immediately assign to the Z matrix those output states which correspond to the eight stable circuit conditions of Table IX, Column (*a*). The result is the partially completed matrix of Table X.

TABLE X.—*Illustrating Development of the Z Matrix.*

(*a*) Partial *Z* Matrix

y: \ *x*:	00	01	10	11
00	01	00		
01	10		00	
10		11		01
11			11	10

(*b*) Completed *Z* Matrix

y: \ *x*:	00	01	10	11
00	01	00	00	01
01	10	00	00	10
10	01	11	11	01
11	10	11	11	10

If we can honestly say that we are concerned with the output state *only* after all the secondary relays of the switching circuit become stable, then any choice whatsoever may be made in the output states which are used to complete the *Z* matrix.

However, there are certain advantages to completing the matrix as we have indicated in Table X, Column (*b*). Here we have assigned to each entry in *Z* the output state associated with the corresponding entry in *F*, even if the entry is uncircled in *F*. For example, both the (01;00) and (01;01) positions in *Z* are given the output state "00," since these correspond to those positions in *F* which contain "3" as an entry, and since the circuit condition "3" is associated with the output state "00." If *Y* is completed in this way, we assure that, as the input state is changed from its present value to a new value, the output state immediately becomes and remains constant at the value corresponding to the ultimate circuit condition.

TABLE XI.—*The Derived Tables of Combinations.*

x_1	x_2	y_1	y_2	Y_1	Y_2	Z_1	Z_2
0	0	0	0	0	0	0	1
0	0	0	1	0	1	1	0
0	0	1	0	0	0	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	0	1	0	1	1
0	1	1	1	1	0	1	1
1	0	0	0	0	1	0	0
1	0	0	1	0	1	0	0
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	0	1	0	0	1
1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	1
1	1	1	1	1	1	1	0

The Tables of Combinations and the Final Circuit

We may reverse the steps taken in the section on analysis and decompose the Y and Z matrices—listing the entries in tables of combinations instead of in the matrix form. The data in Table XI result from the breaking down of the entries found in Tables IX(c) and X(b).

The purpose of this paper is to demonstrate that the problem of the design of sequential relay circuits may be reduced to the problem of the design of combinational-type contact networks. Our task is then complete, and the designer of combinational circuits may now take the data in Table XI and design corresponding secondary relay controlling networks and the output networks. One possible realization of these is given in the final circuit diagram of Fig. 7.

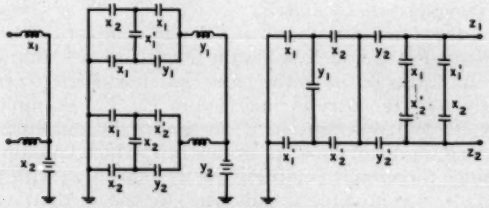


FIG. 7. The synthesized "delay-line" circuit.

IV. FLOW TABLE MANIPULATION

Simplification of the Original Problem Statement

In our zest for listing all possible circuit actions in a problem specification, it is conceivable that we may say more than we actually need to.

TABLE XII.—*The Simplification of a Problem Statement.*

(a) x: 00	01	10	11	Output State	(b) x: 00	01	10	11	Output State	(c) x: 00	01	10	11	Output State
(1)	7	2	6	0	(1)	7	2	6	0	(1)	7	2	6	0
1	4	(2)	3	0	1	4	(2)	3	0	1	4	(2)	3	0
1	4	5	(3)	1	1	4	5	(3)	1	1	4	5	(3)	1
1	(4)	5	6	0	1	(4)	5	6	0	1	(4)	5	6	0
1	4	(5)	6	0	1	4	(5)	6	0	1	4	(5)	6	0
1	4	5	(6)	0	1	4	5	(6)	0	1	4	5	(6)	0
1	(7)	5	8	0	1	(7)	5	3	0	1	(7)	5	3	0
1	4	5	(8)	1	1	4	5	(3)	1					

Two or more circuit conditions which we have listed separately in a flow table might, if proper means of analysis were available, be recognized as a single condition. The elimination of this redundancy in the problem statement may be quite important to us if our object is to get as simple a circuit realization as possible. In the following sections we show that this overstatement of a problem can occur, and we shall give rules for detecting it.

As an example to show how redundancy in a flow table might arise, let us examine Table XII, Column (*a*) and the corresponding output data. This table specifies a circuit which delivers an output ground only at the end of the input sequence "00" → "10" → "11" or at the end of the input sequence "00" → "01" → "11." The first sequence corresponds to the sequence of circuit conditions "1" → "2" → "3," and the second sequence to the sequence of circuit conditions "1" → "7" → "8." The word specification of the problem is satisfied in the flow table, but we may suspect that "3" and "8" could actually be considered as the same circuit condition, since each of them is associated with the same input state and the same output state.

Let us assume, tentatively, that conditions "3" and "8" are equivalent. If this is true, any possible sequence of input states must give identical outputs whether we start with the circled entry "3" or with the circled entry "8." The third and eighth rows of Column (*a*) of Table XII tell us that given changes of input state result in the same ultimate circuit conditions, whether we start initially from "3" or from "8"; and so our original assumption is valid.

Recognition of the equivalence of conditions "3" and "8" allows us to reduce the number of circled entries in the flow table by replacing the designator "8" with the designator "3." If this is done everywhere in the table, Column (*b*) is the result. In this column the eighth row is now redundant and it may be eliminated, as in Table XII, Column (*c*). The effect of this procedure can be expressed alternatively by saying that we have merged rows three and eight, and that the merger was valid because circuit conditions "3" and "8" were equivalent.

Of course we may continue to condense the flow table by merging the fourth, fifth, and sixth rows into a single row. These latter mergers, however, would not change the number of circled entries in the table.

Now that our appetite for simplifying flow tables has been whetted, we may be tempted to see equivalence when it does not exist. In Table XII, Column (*c*), for instance, circled entries "4" and "7" are both associated with the same input state and both give the same output state. If the initial circuit condition is "4," a change of input state from "01" to "11" produces the condition "6." On the other hand, if the initial circuit condition is "7," the same change of input state (from "01" to "11") gives the condition "3." But conditions "3" and "6" cannot be equivalent because they give different output states. Therefore "4" and "7" cannot be considered equivalent either.

TABLE XIII.—*The Simplification of a Problem Statement.*

(*a*)

x: 00	01	10	11	Output State
6	(1)	4	7	10
6	(2)	3	7	10
5	1	(3)	8	11
5	2	(4)	8	11
(5)	2	3	7	11
(6)	1	3	8	00
5	1	4	(7)	00
6	2	4	(8)	01

(*b*)

x: 00	01	10	11	Output State
6	(1)	3	7	10
6	(1)	3	7	10
5	1	(3)	8	11
5	1	(3)	8	11
(5)	1	3	7	11
(6)	1	3	8	00
5	1	3	(7)	00
6	1	3	(8)	01

(*c*)

x: 00	01	10	11	Output State
6	(1)	3	7	10
5	1	(3)	8	11
(5)	1	3	7	11
(6)	1	3	8	00
5	1	3	(7)	00
6	1	3	(8)	01

Sometimes the reasoning is not so straightforward as it was in the above example. For instance, in Table XIII, Column (*a*), we might examine for equivalence the entries "1" and "2." If we assume for the time being that they are equivalent, then examination of the flow table indicates that the validity of our assumption depends on whether or not entries "3" and "4" are equivalent. When we look at the third and fourth rows, we find in turn that "3" and "4" are equivalent if "1" and "2" are. But this follows from the original assumption. Similar reasoning would hold if we were to assume initially the equivalence of "3" and "4." Our conclusions must be: "1" and "2" are equivalent, and so are "3" and "4."

Because of the equivalences discovered above, we may condense Column (*a*) of Table XIII by merging the first and second rows and by merging the third and fourth rows. The procedure is first to replace "2" by "1" and "4" by "3" everywhere in the table (see Column (*b*)) and then to eliminate the redundant rows as in Column (*c*).

Statement of Rules

With the aid of reasoning which we have done above, we now define equivalence as follows:

Two circled entries of a flow table may be considered equivalent if, and only if,

1. each entry appears in the same column of the flow table (is associated with the same input state), and if
2. each entry is associated with the same output state, and if,
3. for each sequence of input states which could start equally well from either of the two circuit conditions involved, the corresponding output sequence is independent of which circuit condition was used as the starting point.

Another way of saying this same thing is this: In a flow table, two circled entries which are associated with the same input state and with the same output state are equivalent *unless* there exists some sequence of input states which may start from either of these two circled entries, and yet which gives corresponding output sequences which differ from each other.

There will be further illustration of these qualifications in the following sections.

Problem Statements Which Have a Unique Simplification

Input-Output Sets

The circled entries of a flow table may be partitioned into mutually exclusive *input-output sets*. Within each such set the members all have a given input state and a given output state in common. If two entries belong to different input-output sets, then either their input states or their output states, or both, differ. For example, in Table XIV, Column (*a*) we may place the entries into the following input-output sets:

(1,7,9,12); (5,14); (2,6,11); (4,8,13); and (3,10,15).

Notice that the pattern of vacancies in the rows corresponding to the members of each set is the same. For instance, the second, sixth, and eleventh rows have the following patterns of entries:

5	②	3
5	⑥	10
5	⑪	3

In each of these rows the position in the third column is left vacant.

For flow tables which are constructed in such a way that the vacancy pattern in the rows corresponding to the members of any given input-output set is the same for all members of the set, there is a unique way of eliminating redundancy and therefore a unique minimum-circled-entry form of the table. This statement will be proved later in the section entitled *Equivalence-Sets* (see page 185).

Meanwhile, let us note that a vacancy in a row of a flow table may indicate one of two things: Either (1) the input transition corresponding to the vacancy is impossible, or (2) for this transition we do not care to define the resulting circuit action. In the latter case we must be satisfied with whatever circuit action actually occurs when the relay circuit is in operation.

When we have a flow table before us, we need not be concerned with the reasons for vacancies—only that they exist. Therefore, for purposes of the following discussion, let us assume that vacancies in a flow table indicate that the corresponding transition in input state is impossible.

Testing for Equivalence

Imagine now that we have before us two identical relay switching circuits of the kind described by a given flow table. Assume also that this flow table meets the requirement that the same vacancy pattern shall exist for all members of a given input-output set. If we wanted to test two different circuit conditions, "i" and "j," for equivalence, we might do this by putting the two switching circuits into the conditions "i" and "j," respectively.

Imagine now that a given sequence of input states is impressed upon each of the circuits. If, at any time in this sequence, we find that an input transition which is possible for one circuit is not possible for the other, we might immediately conclude that the respective conditions of the two circuits are not listed in the same input-output set. Therefore we would have found one sequence of input states which, starting with the initial circuit conditions "i" and "j," did not result in the same output sequences. We should then conclude that "i" and "j" are not equivalent.

TABLE XIV.—*A Unique Simplification of a Problem Statement.*

(*a*)

x: 00	01	10	11	Output State
(1)	11	4	10	01
5	(2)		3	11
5	2	13	(3)	11
12		(4)	15	00
(5)		8		10
14	(6)		10	11
(7)	6	8	3	01
7		(8)	3	00
(9)	11	13	10	01
12	6	13	(10)	11
5	(11)		3	11
(12)	2	4	15	01
1		(13)	10	00
(14)		8		10
1	6	4	(15)	11

(*b*)

x: 00	01	10	11	Output State
(1)	2	4	10	01
5	(2)		3	11
5	2	4	(3)	11
1		(4)	10	00
(5)		8		10
5	(6)		10	11
(7)	6	8	3	01
7		(8)	3	00
(1)	2	4	10	01
1	6	4	(10)	11
5	(2)		3	11
(1)	2	4	10	01
1		(4)	10	00
(5)		8		10
1	6	4	(10)	11

(*c*)

x: 00	01	10	11	Output State
(1)	2	4	10	01
5	(2)		3	11
5	2	4	(3)	11
1		(4)	10	00
(5)		8		10
5	(6)		10	11
(7)	6	8	3	01
7		(8)	3	00
1	6	4	(10)	11

With the aid of the reasoning above, we know that if any two circuit conditions are equivalent to a third, then any input sequence which is possible starting from any one of these conditions is possible starting from the other two, and corresponding output sequences are the same for all. It follows that each of the three conditions is equivalent to the others.

Equivalence-Sets

By the logical extension of the argument above, we know that, for a flow table meeting our restriction of uniform vacancy pattern for all members of an input-output set, the input-output sets themselves may be partitioned into *equivalence-sets*. These equivalence-sets are to be defined in such a way that each member of the set is equivalent to every other member of the set, and so that no member of one equivalence-set is a member of another.

For Column (*a*) of Table XIV, we may—by techniques discussed in preceding sections—determine that the equivalence-sets are: (1,9,12); (7); (5,14); (2,11); (6); (4,13); (8); (3); and (10,15).

It is easiest to test for equivalence when a flow table is in its *primitive* form: that is, with one circled entry per row. If we have determined that the flow table meets our required restrictions on the placement of vacancies, we may place the entries in equivalence-sets. If our object is to simplify the problem statement as far as possible (reduce the number of circled entries to a minimum), the best we can do is to replace each member of an equivalence-set by a single entry from that set, and then to eliminate the resulting superfluous rows of the table. This minimum number of circled entries is clearly unique, and is the same as the number of equivalence-sets.

In Column (*b*) of Table XIV, we have replaced the members of each equivalence-set by the member of this set which has the lowest numerical designation, and then we have eliminated the redundant rows. The table we have derived (Column (*c*)) is the simplest possible way of stating the terminal action specified by the original flow table of Column (*a*).

A Problem Simplification Which is not Unique

If a circuit is described by a primitive flow table which does not meet the requirements of uniformity of vacancy pattern within each input-output set, then it may not be possible to partition the entries of the flow table into equivalence-sets. Practical situations in which there may not be this required uniformity may occur if the input is the output of another sequential switching circuit, or if a human being arbitrarily restricts the input transitions allowable in accordance with his knowledge of the internal state of the circuit.

TABLE XV.—*Two Simplifications of the Same Problem Statement.*

(*a*)

x: 00	01	10	11	Output State
(1)	3		5	01
(2)	7		8	01
6	(3)	4	5	00
1		(4)		10
9		4	(5)	11
(6)	3	4		01
9	(7)		8	11
2	3		(8)	01
(9)		4	8	01

(*b*)

x: 00	01	10	11	Output State
(1)	3	4	5	01
(2)	7	4	8	01
1	(3)	4	5	00
1		(4)		10
2		4	(5)	11
2	(7)		8	11
2	3		(8)	01

(*c*)

x: 00	01	10	11	Output State
(1)	3		5	01
(2)	7		8	01
6	(3)	4	5	00
1		(4)		10
6		4	(5)	11
(6)	3	4	8	01
6	(7)		8	11
2	3		(8)	01

In Column (*a*) of Table XV, the flow table does not meet the restrictions of uniformity of vacancy pattern; see, for example, the first and sixth rows. Inspection of the flow table and the output data in Column (*a*) shows us that the only input sequence which may start with both condition "1" and condition "6" is that beginning "00, 01, ···," and that for this sequence, condition "3" follows immediately whether we start with "1" or with "6." Thus "1" and "6" have a sort of equivalence. We shall call it *pseudo-equivalence* in order to distinguish it from the true equivalence, which we discussed earlier (see section beginning on page 183).

By reasoning similar to that used above, we may establish that entries "2" and "9" possess this pseudo-equivalence also. And the same is true for entries "6" and "9." The pseudo-equivalences which we have found (there are no more equivalences of any kind) in the flow table of Column (*a*) are summarized here.

"1" ↔ "6" "2" ↔ "9" "6" ↔ "9"

Notice that we cannot form mutually exclusive equivalence-sets as we did with the tables meeting the uniform vacancy requirement.

The inability to form sets of equivalent entries leads to situations which were not possible for flow tables meeting the uniform vacancy restriction. If we replace "6" by "1" and "9" by "2" in Column (*a*) of Table XV, and if we then merge the pairs of rows which contained these circled entries, the flow table of Column (*b*) can be derived. (When we compare, for example, the first row of Column (*b*) with the first and sixth rows of Column (*a*), we find that we have retained the information in the two component rows.) If, instead, we replace "9"

by "6" everywhere in Column (*a*), and then merge the rows containing these circled entries, Column (*c*) follows.

Now consider Columns (*b*) and (*c*) of Table XV, both of which represent the same problem, and in neither of which can the number of circled entries be reduced further. It might well be that two different people would have stated the problem in these two forms in the first place, and so we cannot guarantee that there is a unique "simplest" primitive form of the flow table for the general sequential circuit problem.

Row Mergers

An Example in Which the Flow-Table Condensation is not Unique

Column (*a*) of Table VIII had a unique minimum-row form in which no further row mergers were possible. In general, however, when a flow table has "vacancies" there may not be a unique minimum-row form. In the flow table of Column (*a*) of Table XVI, for example,

TABLE XVI.—*Illustrating Non-Unique Flow-Table Condensations.*

(*a*)

x: 00	01	10	11
(1)	2	5	
1	(2)		4
6		(3)	8
	2	3	(4)
1		(5)	4
(6)	2	3	
1	(7)		8
	7	3	(8)

(*b*)

x: 00	01	10	11
(1)	(2)	(5)	4
6	7	(3)	(8)
(6)	2	3	(4)
1	(7)		8

(*c*)

x: 00	01	10	11
(1)	2	(5)	4
1	(2)	3	(4)
(6)	2	(3)	8
1	(7)	3	(8)

let us assume that the output states associated with the various circuit conditions are such that no redundancies exist, and therefore that no two circled entries are equivalent. In this table, any one of the following pairs of rows may be merged:

1, 2 1, 5 2, 4 2, 5 3, 6 3, 8 4, 6 7, 8

If we merge rows one, two, and five; rows three and eight; and rows four and six; the result is that shown in Column (*b*). If, instead, we merge rows one and five, rows two and four, rows three and six, and rows seven and eight, the resulting condensed flow table is that of Column (*c*).

The flow tables of Columns (*b*) and (*c*) in Table XVI do not have rows which can be merged further; yet each contains the same information that the table of Column (*a*) does. From this simple example, therefore, we have demonstrated that, in general, a flow table cannot—by row merging—be reduced to a unique minimum-row form.

Requirements for a Unique Flow-Table Condensation

The result obtained in the previous section leads us naturally to the question: "Are there flow tables in which the operation of row merging leads us to a unique minimum-row form of the table?" The answer is "Yes." If a flow table has no "vacancies" (that is, if no input transitions are prohibited), there is such a unique form. We shall see below the reasoning behind our affirmative answer.

For any flow table of the "completely filled" type, we may form mutually exclusive sets of rows in which the pattern of entries in the rows of each set is the same. (The first three rows in Column (*a*) of Table XVII comprise such a set.) Then all the rows within each set

TABLE XVII.—*Illustrating a Unique Flow-Table Condensation.*

(*a*)

x: 00	01	10	11
(1)	3	2	8
1	3	(2)	8
1	(3)	2	8
5	7	2	(4)
(5)	7	2	4
1	3	(6)	4
9	(7)	6	8
9	7	6	(8)
(9)	7	6	8

(*b*)

x: 00	01	10	11
(1)	(3)	(2)	8
(5)	7	2	(4)
1	3	(6)	4
(9)	(7)	6	(8)

may be merged to give a single composite row which will have the same entry pattern as the members of the set. After all the rows within each set have been merged, the resultant condensed flow table has as few rows as we can obtain. This minimum-row result is unique, and independent of the order in which the mergers were made.

By way of illustration of the principles stated above, consider Column (*a*) of Table XVII. The entry patterns present in the various rows of the table allow the following mutually exclusive sets of rows to be formed: (1,2,3); (4,5); (6); and (7,8,9). The minimum-row form

of this table is found by merging all rows of a set with each other. The result in our present example is the flow table shown in Column (*b*) of Table XVII.

Row Splitting

For the sake of completeness we must include among the possible manipulations on a flow table a process which is the opposite of row merging. It will be called *row splitting*. Future investigation may show that this process is sometimes useful in modifying a flow table so that the assignment of secondary relay states is made easier.

When a row of a flow table is split into several other rows, the pattern of entry designations in each of these new rows must be equivalent to that of the original row. In the flow table of Column (*a*) of Table XVIII, one way of splitting the row containing the circled entries

TABLE XVIII.—*Illustrating Splitting of Rows in a Flow Table.*

(*a*)

x: 00	01	10	11
(1)	6	(5)	3
1	(2)	4	3
7	2	(8)	(3)
(7)	(6)	(4)	3

(*b*)

x: 00	01	10	11
(1)	6	5	3
1	6	(5)	3
1	(2)	4	3
7	2	(8)	(3)
(7)	6	4	3
7	(6)	(4)	3

one and five, and of splitting the row containing the circled entries four and six, is that shown in Column (*b*). In this derived table the rows resulting from the splitting process give the same description of circuit action that the original flow table did, and we could obtain again the original flow table by making all possible mergers in Column (*b*).

Another more complicated type of row splitting occurs if the row we attempt to split has but one circled entry. Then the effect is to increase the number of circled entries in the flow table; or we may say that the single circled entry of such a row is itself split. The rules for *splitting an entry* into two or more parts are easy to state. If we split an entry "i" into m different entries, we take the row of the flow table which contains the circled entry "i" and replace it by m rows which contain circled entries "i_1," "i_2," $\cdots$, "i_m." Each of these m circled entries is to be equivalent to the others. Thus each will be in the same column as the original entry "i," and the output state associated with each will be the same as that of the original entry. In each of the rows of the

new flow table we insert uncircled entries which are either the same as the corresponding entries in the original table or equivalent to them.

By way of illustration, let us, in Column (*a*) of Table XIX, split the entry "1" into three parts, and the entry "3" into two parts. One way of doing this is that given in Column (*b*). Notice that in this latter table the uncircled entries "1" and "3" have been replaced by equivalent

TABLE XIX.—*Illustrating Splitting of Entries in a Flow Table.*

(*a*)

x: 00	01	10	11
(1)	3		4
7	3	(2)	
	(3)	5	4
7	6	5	(4)
1		(5)	
7	(6)	2	
(7)	3		4

(*b*)

x: 00	01	10	11
(1_1)	3_1		4
(1_2)	3_2		4
(1_3)	3_2		4
7	3_2	(2)	
	(3_1)	5	4
	(3_2)	5	4
7	6	5	(4)
1_2		(5)	
7	(6)	2	
(7)	3_1		4

entries everywhere in the table. We could, of course, simplify the table of Column (*b*) to produce again the flow table of Column (*a*).

Combinations of the techniques above may be used to split the rows in a flow table. (See the example leading up to Table XXV.) The test for the validity of such splits is always to determine whether or not the rows resulting from each split may be merged again to give the original row.

(*To be continued.*)

ON H. ALFVÉN'S THEORY OF THE EFFECT OF MAGNETIC STORMS ON COSMIC RAY INTENSITY *

BY

W. F. G. SWANN [1]

ABSTRACT

H. Alfvén's theory envisages a stream of gas shot out from a place on the sun where there is a large magnetic field. The gas, supposed highly conducting, carries the magnetic field H with it, and the field is supposed to be perpendicular to the velocity v. A stationary observer observes an electric polarization perpendicular to H and v, and the potential change across the beam of width h is calculated as Hvh/C. A cosmic ray crossing the beam is supposed to change its energy by $Hohe/C$, and by making H or h large this change of energy can be made as great as is desired. Alfvén calculates a 10 per cent change for the energy of a 3×10^{10} e.v. ray on crossing a beam of width 5×10^{12} cm. carrying a magnetic field of 3×10^{-4} gauss and travelling with a velocity of 2×10^{8} cm./sec.

The present paper recognizes that an observer moving with the beam observes no change of energy, and by a relativity transformation, it is shown that a fixed observer cannot observe a change of energy greater than $2vW/C$, where W is the energy of the cosmic ray. This change amounts to only about 1.4 per cent.

The discrepancy between the conclusions from relativity and those drawn by Alfvén are shown to arise from the fact that the cosmic ray path in the example considered by Alfvén would have, in the beam, a radius of curvature 15 times smaller than the beam width and so could never cross it. Thus relativity gives for the change of energy of the beam an upper limit of the order $2v/C$ times the energy of the beam, and this limit can never be exceeded, however large H or h may be chosen.

An analysis of the considerations resulting from the assumption of a finite conductivity for the space external to the beam shows that the existence of such a conductivity does not change the conclusions.

INTRODUCTION

H. Alfvén[2] has proposed an ingenious theory to explain the effect of magnetic storms on cosmic ray intensity. It is founded upon the assumption that a mass of highly conducting gas originating in a region where there is a large magnetic field emerges as a stream, carrying the magnetic field with it as a result of its high conductivity. The case is considered in which the magnetization is perpendicular to the velocity of the stream, in which case the motion of the matter, in its own magnetic field, brings about a condition in which a stationary observer sees a charge displacement in a direction perpendicular to the velocity and to the magnetic field.[3] Alfvén's theory, as presented, is confined to

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[1] Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] H. ALFVÉN, *Phys. Rev.*, Vol. 75, p. 1732 (1949).

[3] In all that follows, we shall denote by S the so-called laboratory system with respect to which the motion is observed. S' shall denote a system moving with the gas. The velocity, v, shall be parallel to the axis of x. The magnetic field, H, shall be along the axis of z which shall be perpendicular to the plane of the paper, so that the axis of y shall be in the plane of the paper and perpendicular to the velocity.

the case where the stream of gas moves in a non-conducting medium, and our discussion will be limited, in the first instance, to this case. The argument of Alfvén is to the effect that the field along the axis of y is $-vH/C$, and the potential change of a cosmic ray in crossing the beam is vhH/C, where h is the width of the beam and C is the velocity of light. It is thus concluded that if T is the kinetic energy, the change in kinetic energy in crossing the beam is δT, where

$$\delta T = vhHe/C, \tag{1}$$

where e is the charge on the particle.

The change of kinetic energy leads to an expression for the change of cosmic ray intensity, in a manner which does not concern us here,[4] since our problem involves the change in kinetic energy, and in particular the relation (1).

DISCUSSION OF THE THEORY

Relativistic Considerations

In connection with the derivation and implications of Eq. 1, one must observe that we are here dealing with a problem in which the magnetic field moves with the matter. It will be convenient to concentrate our attention upon a finite slab, as represented in Fig. 1.

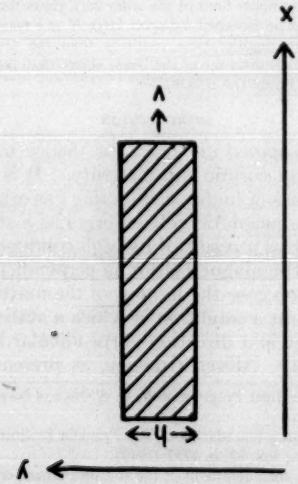

FIG. 1.

[4] Incidentally, there appears to be an error in Alfvén's calculation of the relation between change of intensity and change of energy. This matter is discussed in the present writer's paper "Cosmic Rays," JOUR. FRANKLIN INST., Vol. 251, pp. 120–155 (1951). See in particular pages 146–147.

Now we must observe that in the system S' which moves with the slab, there is no electric field, but only a magnetic field. As a consequence, the cosmic ray particle never gains any energy in terms of measurements made in S'. If W is the energy of the particle[5] and M the momentum, we have that M_x, M_y, M_z, iW/C constitutes a 4-vector. Thus[6] WU_x, WU_y, WU_z, iWC constitutes a 4-vector where U is the velocity of the cosmic ray particle in S. Thus if dashed letters refer to the system S', and if $\beta = (1 - v^2/C^2)^{-\frac{1}{2}}$,

$$W = \beta\left(W' + \frac{vU_x'W'}{C^2}\right). \tag{2}$$

If, in the system S', the particle experiences a change in U_x', from U_{x1}' to U_{x2}', then since W' does not change in S', we shall have a change in W, given by $W_2 - W_1$, where

$$W_2 - W_1 = \frac{\beta W'(U_{x2}' - U_{x1}')v}{C^2}. \tag{3}$$

Now the greatest possible value of $U_{x2}' - U_{x1}'$ is $\pm 2C$. Thus, if $|W_2 - W_1|$ is the absolute value of the change of W, we have

$$|W_2 - W_1| < \frac{2\beta W'v}{C}. \tag{4}$$

From (2)

$$W_1 = \beta W'\left(1 + \frac{vU_{x1}'}{C^2}\right)$$

so that

$$|W_2 - W_1| < \frac{2W_1v}{C\left(1 + \frac{vU_{x1}'}{C^2}\right)}.$$

Now the smallest possible value of $(1 + vU_{x1}'/C^2)$ is $(1 - v/C)$. Hence

$$|W_2 - W_1| < \frac{2W_1v}{C - v}$$

$$|T_2 - T_1| < \frac{2v}{C - v}(T_1 + m_0C^2), \tag{5}$$

and unless v is comparable with C

$$|T_2 - T_1| < \frac{2v}{C}(T_1 + m_0C^2). \tag{6}$$

[5] It is to be observed that $T = W - m_0C^2$.

[6] The essentials of the derivation immediately following are given in my paper cited under footnote 4. See page 134.

Alfvén's Numerical Conclusions

Now Alfvén[2] considers a cosmic ray of energy 3×10^{10} e.v., and demands, through his formula (1) a ten per cent change, with $h = 5 \times 10^{12}$, $v = 2 \times 10^{8}$ cm./sec. He finds $H = 3 \times 10^{-4}$.

With the above value of v, the quantity β is practically unity, and our formula (6) gives

$$\frac{T_2 - T_1}{T_1} < \left(\frac{4}{300}\right)\left(1 + \frac{10^9}{3 \times 10^{10}}\right)$$
$$< 1.37 \times 10^{-2}.$$

Thus, Alfvén, from the aforesaid data, calculates a 10 per cent change in T, and we have shown that the maximum possible change is 1.37 per cent. Moreover, in calculating the value we have not had occasion to utilize any quantity but v. What is the origin of the discrepancy in this matter?

Origin of the Discrepancy

It will be shown that the aforesaid discrepancy lies chiefly in the fact that Alfvén's magnetic field $H = 3 \times 10^{-4}$ would never allow a cosmic ray particle to pass through the slab. The easiest way to see this is from consideration of radius of curvature. Let us consider matters in S'. If ρ is the radius of curvature of our cosmic ray in the field H', we have, if m is the relativistic mass,

$$\frac{mU'^2}{\rho} = \frac{H'eU'}{C}$$

$$\rho = \frac{mU'C}{H'e} = \frac{mC^2}{H'e}\left(\frac{U'}{C}\right) = \frac{W'}{H'e}\left(\frac{U'}{C}\right) \qquad (7)$$

$$\rho < W'/H'e.$$

For[7] $W' = 3 \times 10^{10}$ e.v. and $H = 3 \times 10^{-4}$

$$\rho < \frac{3 \times 10^{10}}{300 \times 3 \times 10^{-4}} = \frac{10^{12}}{3}.$$

Thus the radius of curvature is fifteen times less than the thickness (5×10^{12} cm.) assumed for the slab. A particle entering the slab in S' could not possibly travel a distance greater than 2ρ parallel to the axis of y before returning and emerging from the slab. Thus, no particle can penetrate right through the slab under the conditions cited. No second or third "try" in the event of the particle's seeking re-entry can be any more successful. Thus, the example cited by Alfvén represents a situation impossible of realization.

[7] We need not concern ourselves with the difference between W' and W in the calculation.

It is of interest to trace the above story by another path starting with Alfvén's expression (1), *viz.*

$$\delta T = \frac{vhHe}{C}$$

which we must assume to certainly fail if $h > 2\rho$. Since

$$H_z = \beta\left(H_z' - \frac{v}{e}E_y'\right) = \beta H_z' \tag{8}$$

$$\delta T = \frac{\beta vhH'e}{C}.$$

Now we can substitute for He' in terms of ρ by using (7). Thus

$$\delta T = \frac{\beta W' U' vh}{\rho C^2}.$$

δT increases with increase of h, and its maximum value is reached when $h = 2\rho$, thus

$$\delta T < \frac{2\beta v}{C}\left(\frac{U'}{C}\right) W' < \frac{2v\beta}{C} W' \tag{9}$$

which is the same as the result (4) obtained directly from the relativity transformation.

Connection between Relativistic Formula and Alfvén's Formula when the Latter is Applicable

On the other hand, we can show that if the conditions of smallness of change of energy are satisfied, then Alfvén's result follows from the relativity transformation. Thus if θ_1' is the angle made by the cosmic ray with the plane of the slab at entrance in S' and θ_2' is the angle made at emergence, when emergence occurs, we have, referring to (3)

$$U_{x2}' - U_{x1}' = U' \cos\theta_2' - U' \cos\theta_1'.$$

Now, if ds' is an element of path in the slab

$$dy = \sin\theta' \, ds' = \rho \sin\theta' \, d\theta'$$

where ρ is the radius of curvature, which, in this case, is constant. Thus

$$-h = \rho(\cos\theta_2' - \cos\theta_1') = \frac{\rho}{U'}(U_{x2}' - U_{x1}'). \tag{10}$$

Now

$$\frac{mU'^2}{\rho} = \frac{H'eU'}{C}.$$

Hence, from (10)

$$U_{x2}' - U_{x1}' = \frac{H'eh}{mC} = \frac{H'ehC}{W'}$$

so that (3), which is derived from relativity, yields

$$W_2 - W_1 = \frac{\beta H'ehv}{C}$$

or, since, from (8), $H_z = \beta H'$

$$|T_2 - T_1| = |W_2 - W_1| = \frac{Hehv}{C} \tag{11}$$

which is Alfvén's result limited however, as aforesaid, to an upper limit defined by (5), a limit which amounts to a fraction $2v/C$ of the energy itself, and which is in general much less than 10 per cent. *No assumption of large magnetic field or large value of h can enable* $(W_2 - W_1)$ *to assume a value larger than that determined by* (5).

It is of some interest to inquire how the relativity transformation symbolized by its results in (5) seems to "know" of the impossibility of penetrating the slab under the conditions stated. The fact is that the transformation from the pure magnetic field in S' to the combined electric and magnetic fields E and H in S is one obeying relativity. Also the force equation of electrodynamics which tells us that there is no gain of energy of the particle in S' while there may be a loss or gain in S is also an equation obeying relativity; and if these equations had permitted the passage of the particle through the slab under conditions in which (11) had given a greater value of $W_2 - W_1$ than was permitted by (5), an *algebraical* inconsistency would have been involved.

Case of a Continuous Beam

In Alfvén's paper, he assumes a continuous stream of gas emerging from the sun. If such a gas spreads on its journey, the value of v will vary. This appears at first sight to introduce a complication. However, the complication is obviously trivial as regards its practical effects. Thus, consider any portion of the beam through which the path of a cosmic ray particle is contemplated. If S' applies to this portion of the beam, and if v is its velocity, the motion in S' will be determined by the magnetic field in S' and by an electric field which comes essentially from the charge displacement in other parts of the beam which do not move with the velocity v.[8] The field produced in S' by these displacements, amounting as it does to the field produced by the charges on a "condenser" when evaluated outside of the region between the plates of the condenser, will be negligible for our purposes. A

[8] There is, of course, no charge displacement in S' at the portion moving with velocity v.

formal, exact, analysis of all that is here involved would result in a lengthy presentation whose end point would, in the nature of things, involve but a small correction on the conclusions which would be drawn by ignoring it.

CASE WHERE THE SLAB MOVES IN A CONDUCTING MEDIUM

Limiting Case of Infinite Conductivity

The fact that we have supposed the slab to carry its internal magnetic field with it demands, for logical consistency, that the external medium, if infinitely conducting, shall resist change of magnetic field within itself to the extent that the magnetic field H remains zero everywhere outside of the slab as viewed in the system S. If the conductivity of the region external to the slab is ohmic as viewed in S, then even though in the limit it tends to infinity, no space charge will arise in S, and consequently[9] by the relativity transformation, there will be no space density in S'. The only charge of any kind will be on the surface of the slab. From symmetry the only possible surface charge will be a density[10] σ' at $y = h/2$ and $-\sigma'$ at $y = -h/2$.

The field component E_{y_0}' in S' just outside the slab at its mid-point will be a small negative fraction $-\epsilon$ of the field inside the slab which is $-4\pi\sigma'$. Thus $E_{y_0}' = 4\pi\epsilon\sigma'$.

---

[9] Some care is necessary in drawing this conclusion, as is evidenced by the fact that a system which viewed from S appears as a uniformly magnetized sphere, appears in the other system as having in addition equal and opposite charge densities displaced along a line perpendicular to the magnetic induction in the sphere and to the velocity, v. It will, in fact, be recalled that if U is the vector potential, and ϕ the scalar potential, U, $i\phi$ constitute a 4-vector, so that the zero value of ϕ in one system does not guarantee $\phi' = 0$ for the other system. The *apparent* paradox in relation to the invariance of total charge becomes resolved as follows:

The densities ρ and ρ' for a given sign of electricity, let us say negative, are related in the two systems by

$$\rho_-' = \beta\rho_-\left(1 - \frac{u_{x1}v}{C^2}\right)$$

where u_{x1} is the velocity in the system x.

Suppose now we superpose a positive density ρ_+ in S, such that $\rho_+ + \rho_- = 0$. We shall have

$$\rho_+' = \beta\rho_+\left(1 - \frac{u_{x2}v}{C^2}\right).$$

Then

$$\rho_-' + \rho_+' = \beta\left[\rho_-\left(1 - \frac{u_{x1}v}{C^2}\right) + \rho_+\left(1 - \frac{u_{x2}v}{C^2}\right)\right]$$

$$= \beta\left[\rho_+ + \rho_- - \frac{v}{C^2}\left(\rho_+u_{x2} + \rho_-u_{x1}\right)\right].$$

Hence, if, and only if $(\rho_+u_{x2} + \rho_-u_{x1}) = 0$, can we conclude that $\rho_+' + \rho_-' = 0$ follows from $(\rho_+ + \rho_-) = 0$. In our present problem, since $H = 0$ in S, curl $H = 0$ and the total current density $(\rho_+u_{x2} + \rho_-u_{x1})$ is zero, and the desired result follows.

[10] σ' will be sensibly constant except near the edges of the slab.

If λ_1 is the conductivity within the slab, the y component of the current density at $x = h/2$ will be $-4\pi\sigma'\lambda_1$. The corresponding component outside will be[11] $\left(E_{y0}' + \frac{v}{C}H_{z0}'\right)\lambda_2$. Thus we must have

$$-4\pi\sigma'\lambda_1 = \left(4\pi\sigma'\epsilon + \frac{v}{C}H_{z0}'\right)\lambda_2. \tag{12}$$

Now since H is zero at all points outside the slab as measured in S, we have for a point outside the slab

$$0 = H_z = \beta\left(H_z' - \frac{v}{C}E_y'\right). \tag{13}$$

Thus

$$\frac{v}{C}H_{z0}' = \frac{v^2}{C^2}E_{y0}' = \frac{v^2}{C^2}(4\pi\epsilon\sigma').$$

Thus from (12)

$$-4\pi\sigma'\lambda_1 = 4\pi\epsilon\sigma'\left(1 - \frac{v^2}{C^2}\right)\lambda_2. \tag{14}$$

However, this is a relation between λ_1 and λ_2 which depends upon ϵ and v only, and these quantities have nothing to do with the conductivity of the media.[12] The only possibility is represented by $\sigma' = 0$, which permits (14) to hold with any assignment of the ratio λ_1/λ_2.

If $\sigma' = 0$, we have, of course, $E_x' = E_y' = E_z' = 0$ at all points outside the slab. As a consequence, in the same region $H_z' = 0$ on account of (13), and $H_y' = 0$ because

$$0 = H_y = \beta\left(H_y' + \frac{v}{C}E_z'\right).$$

Moreover, $H_x' = H_x = 0$.

The fact that the state of no electric or magnetic field outside of the slab in either S or S' constitutes a solution of the problem is obvious by direct inspection, since it calls for no currents outside the slab in S' resulting from the motion of that slab. Inside the slab itself there is nothing but a magnetic field, and the whole problem reverts completely to the problem we have already considered when we attributed zero conductivity to the medium outside of the slab. The conclusions are exactly the same.

How Is the Condition of Zero External Field Realized in S'?

It is of interest to inquire how it can come about that the "magnetized" slab can exist in the external medium which itself shows no

[11] Observe that $v_x = -v$.

[12] We again remark that such a relation as (12) has sense as λ_1 and λ_2 both tend to infinity.

magnetic field in either S or S'. The fact is that, in the simple case, the magnetic field in the slab demands a current sheet circulating around its boundary in planes perpendicular to the axis of z. This sheet in itself would call for a return magnetic flux outside the slab, a flux which was small in amount at any point but widely spread out so that its total integrated value was equal to the total flux through the slab. Now the phenomenon which provides for the cancellation of the magnetic field outside the slab is another current sheet flowing in the reverse sense outside the first one and again in planes perpendicular to the axes of z. If these two current sheets have different cross-sectional areas,[13] they can cancel as regards production of magnetic field outside the outermost, but give a resultant uniform field inside the smaller. In the limit, when the currents are infinitely large, their cross-sections can afford to differ by only infinitesimal amounts, and this condition represents the limiting one applicable for infinite conduction in the medium external to the slab.

In case this above picture presents any difficulty, we may cite the particular simple example of a uniformly charged rotating sphere. We shall not trace the details of calculation for this case, which are elementary, and generally known. Such a sphere gives, at points outside, a magnetic potential Ω, where

$$\Omega = \frac{M \cos \theta}{r^2}.$$

Here $M = \sigma\omega a^4$, where σ is the surface density in e.m.u., ω is the angular velocity and a is the radius.

The field inside is uniform and is given by H, where

$$H = \frac{2M}{a^3}.$$

It will thus be seen that if we have two spheres of radii a_1, and a_2 with σ's adjusted so that $M_1 = -M_2$ these spheres will cancel as regards magnetic field outside of the larger, while inside the smaller they will have a resultant magnetic field given by

$$H_1 - H_2 = 2M\left(\frac{1}{a_1^3} - \frac{1}{a_2^3}\right).$$

If $a_1 - a_2$ is small $(= \delta a)$ we may write

$$H_1 - H_2 = \frac{-6M\delta a}{a^4}.$$

[13] We refer here to the cross-section of the whole "solenoid structure" which constitutes such a sheet. This cross-sectional area is, of course, sensibly equal to that of the slab.

Case Where Conductivities λ_1 and λ_2 Are Finite[14]

The complete problem here involved can become one of considerable complexity. The currents arising from the motion will affect the magnetic field, moreover the charge distributions arising on the slab to provide for continuity of current flow across the slab boundaries will bring about x-components of the current density at any rate in regions remote from the central regions and[9] will destroy even the conclusion that the volume charge density shall be zero in S if it is zero in S' and vice-versa.

We shall not attempt an exhaustive solution of the problem, but shall content ourselves with a kind of model founded upon the considerations represented in Eq. 13, but without the restriction that the magnetic field shall be zero in S for points external to the slab.

We have in S a slab with face A at potential $V/2$ and face B at potential $-V/2$, respectively, A and B being at $x = h/2$ and $x = -h/2$, respectively. Outside the slab, the current density is determined entirely by the field arising from this potential difference. Inside the slab we have a force per unit charge given by $E_y - vH_z$. The line integral of the external field from A to B is V. The line integral of the internal total force per unit charge from B to A is $-V - vHh/C$.

Suppose R_e is the ordinary external resistance which would be measured between the two surfaces of the slab, if the slab itself were of zero conductivity, and R_i the resistance which would be measured if the conductivity of the external medium were zero. Then we have

$$I = \frac{V}{R_e} = \frac{-V - vHh/C}{R_i}. \tag{15}$$

Hence,

$$V\left(\frac{1}{R_e} + \frac{1}{R_i}\right) = \frac{-vHh}{CR_i}$$

$$V = -\left(\frac{R_e}{R_i + R_e}\right)\frac{vHh}{C}.$$

The field E inside the slab is

$$E = \left(\frac{R_e}{R_i + R_e}\right)\frac{vH}{C}. \tag{16}$$

This has its largest value $E = vH/C$ when $R_e = \infty$ which represents the case first discussed in this paper and the result given by Alfvén.

[14] The effect of finite conductivity has been discussed in the writer's paper "Cosmic Rays," JOUR. FRANKLIN INST., Vol. 251, pp. 120–155 (1951). The conclusions drawn at the bottom of p. 134 and the top of p. 135 require correction in the light of the development in the present paper.

We have already studied the other extreme where $R_s = 0$ and where there is no magnetic field external to the slab as observed in S. In this case the result $E = vH/C$ also evolves. We can form a quasi-quantative picture of the intermediate as follows:

It is easy to see that the induced currents *external* to the slab tend to reduce the field which would have been present in their absence. They reduce the field not only in the external region, but in the interior of the slab. Thus if the field H in (16) represents the field actually in the slab, it should be multiplied by a factor greater than unity. While we have not traced everything in detail, it is reasonably obvious that the net effect will result in a value of E which is never greater than the value vH/C where H is the actual field in the slab, this result being attained exactly in the two limiting cases where $R_s = \infty$ and where $R_s = 0$, respectively.

New Tractor Sprayer Attachment.—A new tractor sprayer attachment, designed to meet general spraying requirements for pastures, grain fields, fence rows, right of ways, and many other uses, has been developed by The F. E. Myers & Bro. Co., Ashland, Ohio.

Called "Mighty-Mite," the new sprayer attachment is able to cover from 33 to 52 ft. in one swath. It is designed for operating pressures of 30 to 300 lb. and has a maximum pump capacity of 13 gal. per minute.

The manufacturer states that the "Mighty-Mite" attachment is compact, simple in operation, and may be installed or dismounted from the farm tractor with minimum effort and time.

The "Mighty-Mite" comes complete with pump which mounts directly on to the tractor power shaft, discharge manifold, hose and nozzle assembly. Extra attachments, including extra hose, spray gun, rear mounting assembly and extra nozzles, are available.

New Equipment for Demonstrating Fluid Mechanics.—A comprehensive equipment which will expand college mechanical and aeronautical engineering laboratory facilities in the field of fluid mechanics has been announced by the General Electric Company's Apparatus Educational Sales Section.

The packaged device is an adjustable-blade, multistage, axial-flow fan with a cradled d-c. dynamometer. It is designed to help give a better understanding of the fundamentals of energy transfer and fluid flow encountered in axial-flow turbomachinery.

Experiments performed with the unit can be equally applicable to the basic fluid mechanics of the multistage axial-flow compressor used in gas turbines and other applications. It also may be used in dynamometer absorption demonstrations as a prime mover driving other test machines, and as a low pressure air supply of substantial volume for many types of laboratory test work.

The basic G-E unit consists of an axial-flow fan direct-connected to and mounted on a common bedplate with a 7½-hp. d-c. cradled dynamometer. Dynamometer scale readings and speed provide for direct measurement of horsepower required by the fan for all test conditions.

The fan has a maximum speed rating of 3000 rpm., and the dynamometer, equipped with a field rheostat, 4000 rpm. However, as shipped from the factory, the field rheostat has a stop limiting the speed to approximately 3000 rpm. Minimum speed of the dynamometer is limited to approximately 500 rpm. because of inadequate ventilation and cooling at lower speeds.

Variable voltage control for the dynamometer is provided by including a G-E speed variator utilizing 220-volt, 60-cycle, 3-phase current. This power supply unit provides continuously adjustable armature and field control permitting selection of any desired speed setting within the horsepower and speed rating of the dynamometer.

The fan assembly is made up of two stages with 24 identical rotor blades and 37 identical stator blades in each stage. All blades can be rotated 360 deg. and a protractor-type template is furnished for accuracy in adjusting blade angles.

Rotor blades are individually cast of aluminum alloy. Stator blades are made of sheet steel formed to the proper shape.

INTERPRETATION OF THE AMPÈRE EXPERIMENTS

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

SUMMARY

The celebrated Ampère experiments and their phenomenological interpretation by Ampère's equation are, in many respects, more closely allied with the spirit of modern physics than are the flux visualizations of Faraday and Maxwell. Further study of this subject may lead to an improved theory of electrodynamics.

Ampère's equation for the force between current elements is unique, assuming the correctness of Ampère's five postulates. But two of these postulates may be questioned. The present paper develops the general equations that apply if either the fifth or the third postulate is omitted.

1. INTRODUCTION

The discovery that an electric current can deflect a magnetic needle was announced by Hans Christian Oersted (1) [3] in July, 1820. On September 11 of that year, Oersted's discovery was described before the French Academy by Arago; and only a week later André-Marie Ampère showed that two current-carrying wires exert a force on each other. Ampère's celebrated researches were summarized in his paper (2), "Mémoire sur la théorie mathématique des phénomènes électrodynamiques, uniquement déduite de l'expérience." Maxwell said of this paper, "It is perfect in form and unassailable in accuracy; and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics."

The phenomenological approach of Ampère was continued by Gauss (3), Grassmann (4) (1845), Stefan (5) (1869), Tait (6) (1873), Helmholtz (7) (1873), Riemann (8) (1876), and Korteweg (9) (1881), all of whom treated electrodynamics without the concept of a magnetic field. Maxwell gave a good résumé of this work in Chapter 23 of his *Electricity and Magnetism* (1873). Had he chosen to continue in the direction mapped out by the foregoing investigators, the present status of electromagnetism might be vastly different from what it is. Actually, Maxwell preferred the Faraday visualization in terms of flux lines; and the resulting mathematical edifice was so imposing that pre-Maxwellian work on electrodynamics was soon almost forgotten.

Yet there are at least two reasons why Ampère's work should not be neglected:

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

1. A direct calculation of force between electric circuits, without the intermediary of a magnetic field, may yield results very simply, free from the uncertainties that are sometimes associated with the magnetic-field concept (10).
2. An application of the Ampère method to moving charges is a promising step toward an improved theory of electrodynamics (11).

The textbooks list two forms of equation based on Ampère's experiments. The first equation is that used by Ampère himself. Expressed in rationalized mks units, it may be written

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{\mu I_1 I_2 ds_1 ds_2}{4\pi r^2} [2 \sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2], \qquad (1)$$

where $d^2\mathbf{F}_2$ is the force exerted by current element $I_1 ds_1$ on element

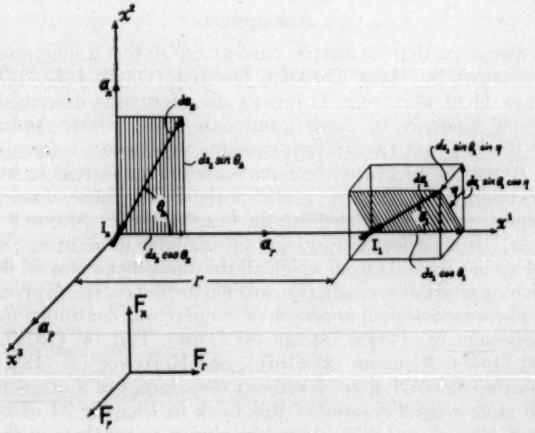

FIG. 1.

$I_2 ds_2$ (Fig. 1), and $\mathbf{a}_r$ is a unit vector in the positive direction along the x^1-axis. The equation may be written also as

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{\mu I_1 I_2}{4\pi r^2} [2(d\mathbf{s}_1 \cdot d\mathbf{s}_2) - 3(d\mathbf{s}_1 \cdot \mathbf{a}_r)(d\mathbf{s}_2 \cdot \mathbf{a}_r)]. \qquad (1a)$$

A characteristic of Ampère's equation is that the force is always along the line of centers.

The other equation seems to have been first published by Grassmann (4) in 1845 and independently developed by Reynard (6) in 1870

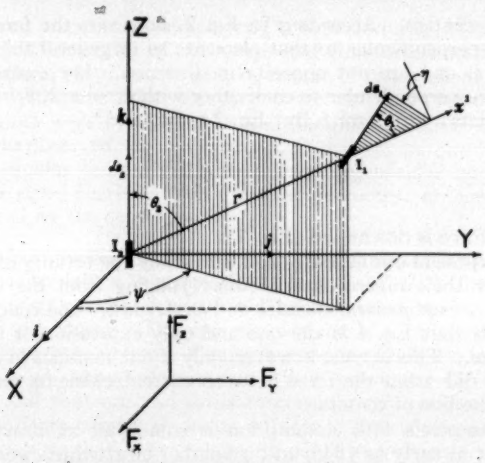

FIG. 2.

and Tait in 1873. In vector notation, it is

$$d^2\mathbf{F}_2 = \frac{\mu I_1 I_2}{4\pi r^2}[(d\mathbf{s}_1 \times \mathbf{a}_r) \times d\mathbf{s}_2.] \tag{2}$$

Written in cartesian form, Eq. 2 is

$$\begin{cases} d^2\mathbf{F}_r = \dfrac{\mu I_1 I_2 ds_1 ds_2}{4\pi r^2} \sin\theta_1 \sin\theta_2 \cos\eta, \\ d^2\mathbf{F}_n = -\dfrac{\mu I_1 I_2 ds_1 ds_2}{4\pi r^2} \sin\theta_1 \cos\theta_2 \cos\eta, \\ d^2\mathbf{F}_p = -\dfrac{\mu I_1 I_2 ds_1 ds_2}{4\pi r^2} \sin\theta_1 \cos\theta_2 \sin\eta, \end{cases} \tag{2a}$$

where the nomenclature is as in Fig. 2. Modern treatments of Eq. 2 are given by Mason and Weaver (12), Frank (13), Harnwell (14), and others (15).

Most physicists do not seem to realize that Eq. 2 is not the equation of Ampère and cannot be derived from it. In fact, the force given by Eq. 2 is not even along the line of centers, although both Eq. 1 and 2 satisfy Ampère's experimental results.

According to Eq. 1, the force on element 1 is always equal and opposite to the force on element 2, and these forces are always along

the line of centers. According to Eq. 2, however, the force on each element is perpendicular to that element; so in general the forces are not along $\mathbf{a}_r$ and are not opposite in direction. For example, if the elements are perpendicular to each other with $\theta_1 = \pi/2$, $\theta_2 = 0$, $\eta = 0$, Eq. 1 predicts a *zero force*. But Eq. 2 reduces to

$$d^2\mathbf{F}_2 = -\mathbf{a}_r \frac{\mu I_1 I_2 ds_1 ds_2}{4\pi r^2},$$

where the force is downward (Fig. 1).

The purpose of our investigation is to study the totality of equations that satisfy the Ampère experiments. Judging from the title of his paper, ". . .—*uniquement* déduite de l'expérience," one concludes that Ampère felt that Eq. 1 is the one and only expression of the experimental facts. This is true, however, only if one assumes in addition—as Ampère did—that the force between current elements must always act along the line of centers.

That Ampère's fifth assumption is somewhat arbitrary (4) was pointed out as early as 1845, and a number of attempts were made to obtain more general expressions. In particular, Stefan (5) derived an equation containing two arbitrary constants and subsuming both the Ampère and the Grassmann equations as degenerate cases. Stefan's equation is the most general one obtainable from the Ampère experiments with the additional assumption that the force on ds_2 is in the plane determined by $d\mathbf{s}_1$ and $\mathbf{a}_r$. Korteweg (9) included the possibility of torques on current elements. Each element is considered as analogous to a tiny bar magnet, which can not only move bodily but can turn about an axis. Since an isolated current element is impossible, however, one finds it difficult to see how such torques could have physical significance.

2. THE AMPÈRE EXPERIMENTS AND POSTULATES

Ampère made four null experiments on closed electric circuits in air. To detect forces, he employed a suspended loop of wire, so arranged that it did not respond to the earth's magnetic field. Further experimental work, extending over more than a century, has not revealed any error in Ampère's experimental results.

The four experiments (2) were as follows:

1. A conductor was doubled back on itself. Current in this conductor produced no measurable effect on the suspended coil.
2. Two parallel conductors carried the same current. One conductor was straight; the other was of the form ⊐⊏⊐⊏⊐. When the detecting coil was midway between the two conductors, it experienced no force.

3. The third experiment employed a wire forming a circular arc, pivoted at its center of curvature and counterbalanced. Contact was made to the movable wire by means of two stationary mercury cups. With the wire carrying current, other current-carrying conductors were brought near the apparatus; but no tendency to turn was observed.
4. Three circular coaxial loops were used, all parallel and traversed by the same current. With similar geometry, no force was experienced by the center loop.

These four experiments dealt with closed circuits. But Ampère wished to formulate a mathematical expression for the force between *current elements*, so that forces could be calculated by integration for circuits of any shape. The current element is a convenient concept but it is also a pure fiction. Common sense is as helpless in predicting the behavior of current elements as it is in predicting the behavior of unicorns. All that one can do is to consider the various expressions for force between current elements, subject to the requirement that the complete circuits shall behave as in Ampère's experiments.

Corresponding to the four experiments, it is customary to infer the following:

Postulate I. The force between two current elements is directly proportional to the product of the currents and is unchanged in magnitude but reversed in direction if either current is reversed.

Postulate II. An elementary current-carrying conductor $d\mathbf{s}$ may be replaced by its components dx^1, dx^2, dx^3 along three axes, without altering the force on another current element.

Postulate III. A *closed* circuit (1) produces no component of force *along* the current element $d\mathbf{s}_2$.

Postulate IV. The force between current elements $I_1 ds_1$ and $I_2 ds_2$ is directly proportional to the lengths ds_1 and ds_2 and is inversely proportional to the square of the distance between the elements.

These postulates are in agreement with Ampère's experiments, but they are not unique and they are not sufficient to determine a force equation.

3. FORCE BETWEEN CURRENT ELEMENTS

Consider two elementary conductors with lengths ds_1 and ds_2, carrying currents I_1 and I_2 (Fig. 1). The conductors are represented by the vectors $d\mathbf{s}_1$ and $d\mathbf{s}_2$, the sense being in the directions of the currents. Without loss of generality, we can introduce a cartesian coordinate system so that $d\mathbf{s}_2$ is at the origin and lies in the x^1x^2-plane; and the line of centers is along the x^1-axis. The element $d\mathbf{s}_1$ lies in a plane making an angle η with the x^1x^2-plane.

According to Postulate II, the vectors $d\mathbf{s}_1$ and $d\mathbf{s}_2$ may be replaced by vector sums along the axes:

$$d\mathbf{s}_1 = \mathbf{a}_r(ds_1 \cos\theta_1) + \mathbf{a}_n(ds_1 \sin\theta_1 \cos\eta) + \mathbf{a}_p(ds_1 \sin\theta_1 \sin\eta),$$
$$d\mathbf{s}_2 = \mathbf{a}_r(ds_2 \cos\theta_2) + \mathbf{a}_n(ds_2 \sin\theta_2),$$

where $\mathbf{a}_i$ are unit vectors. It will be convenient to designate by d^2F_{ijk} the kth component of the force exerted by the ith component of $I_1 d\mathbf{s}_1$ on the jth component of $I_2 d\mathbf{s}_2$. By Postulates I and IV, d^2F_{ijk} is directly proportional to the product of the ith component of $I_1 d\mathbf{s}_1$ and the jth component of $I_2 d\mathbf{s}_2$. By Postulate IV, d^2F_{ijk} is inversely proportional to r^2. Thus the force components may be written

$$\begin{cases} d^2F_{11k} = \dfrac{A_{11k}}{r^2}|I_1||I_2|ds_1 ds_2 \cos\theta_1 \cos\theta_2, \\ d^2F_{21k} = \dfrac{A_{21k}}{r^2}|I_1||I_2|ds_1 ds_2 \sin\theta_1 \cos\theta_2 \cos\eta, \\ d^2F_{31k} = \dfrac{A_{31k}}{r^2}|I_1||I_2|ds_1 ds_2 \sin\theta_1 \cos\theta_2 \sin\eta, \\ d^2F_{12k} = \dfrac{A_{12k}}{r^2}|I_1||I_2|ds_1 ds_2 \cos\theta_1 \sin\theta_2, \\ d^2F_{22k} = \dfrac{A_{22k}}{r^2}|I_1||I_2|ds_1 ds_2 \sin\theta_1 \sin\theta_2 \cos\eta, \\ d^2F_{32k} = \dfrac{A_{32k}}{r^2}|I_1||I_2|ds_1 ds_2 \sin\theta_1 \sin\theta_2 \sin\eta, \\ \qquad\qquad k = 1,2,3. \end{cases} \tag{3}$$

If both elements are directed along the x^1-axis, there is no physical way of distinguishing the directions perpendicular to this axis. Consequently,

$$A_{112} = A_{113} = 0. \tag{4}$$

Also, by symmetry,

$$A_{311} = A_{211},\ A_{313} = A_{212},\ A_{213} = A_{312}. \tag{5}$$

There remain thirteen arbitrary coefficients A_{ijk} in Eq. 3. As shown in Section 4, however, some of these coefficients can be eliminated by employing Postulate III.

It is now advantageous to introduce a new coordinate system (X, Y, Z), so oriented that $d\mathbf{s}_2$ lies along the Z-axis (Fig. 2). Evidently the force on $I_2 d\mathbf{s}_2$, in the direction of its axis, is

$$d^2F_Z = (d^2F_{111} + d^2F_{211} + d^2F_{311} + d^2F_{121} + d^2F_{221} + d^2F_{321}) \cos\theta_2 + (d^2F_{212} + d^2F_{312} + d^2F_{122} + d^2F_{222} + d^2F_{322}) \sin\theta_2$$

or

$$d^2F_Z = \frac{|I_1||I_2|ds_1ds_2}{r^2}[A_{111}\cos\theta_1\cos^2\theta_2 + A_{211}\sin\theta_1\cos^2\theta_2\,(\sin\eta+\cos\eta) + A_{121}\cos\theta_1\sin\theta_2\cos\theta_2 + (A_{221}+A_{212})\sin\theta_1\sin\theta_2\cos\theta_2\cos\eta + (A_{321}+A_{312})\sin\theta_1\sin\theta_2\cos\theta_2\sin\eta + A_{122}\cos\theta_1\sin^2\theta_2 + A_{222}\sin\theta_1\sin^2\theta_2\cos\eta + A_{322}\sin\theta_1\sin^2\theta_2\sin\eta]. \quad (6)$$

But from the geometry of Fig. 2,

$$\sin\theta_2 = q/r, \qquad \cos\theta_2 = Z/r,$$
$$\sin\psi = Y/q, \qquad \cos\psi = X/q,$$
$$ds_1\cos\theta_1 = \frac{X}{r}dX + \frac{Y}{r}dY + \frac{Z}{r}dZ,$$
$$ds_1\sin\theta_1\cos\eta = -\frac{XZ}{rq}DX - \frac{YZ}{rq}dY + \frac{q}{r}dZ,$$
$$ds_1\sin\theta_1\sin\eta = \frac{Y}{q}dX - \frac{X}{q}dY,$$

where dX, dY, dZ are the components of $d\mathbf{s}_1$ along the three axes, and $q = (X^2+Y^2)^{\frac{1}{2}}$. Substitution into Eq. 6 gives

$$d^2F_Z = |I_1||I_2|ds_2[P_X dX + P_Y dY + P_Z dZ], \quad (7)$$

where

$$P_X = (A_{111}-A_{221}-A_{212})\frac{XZ^2}{r^5} + (A_{121}-A_{222})\frac{qXZ}{r^5} + (A_{312}+A_{321})\frac{YZ}{r^4} - A_{211}\frac{XZ^2}{qr^3} + A_{211}\frac{YZ^2}{qr^4} + A_{122}\frac{q^2X}{r^5} + A_{322}\frac{qY}{r^4},$$

$$P_Y = (A_{111}-A_{221}-A_{212})\frac{YZ^2}{r^5} + (A_{121}-A_{222})\frac{qYZ}{r^5} - (A_{312}+A_{321})\frac{XZ}{r^4} - A_{211}\frac{YZ^2}{qr^3} - A_{211}\frac{XZ^2}{qr^4} + A_{122}\frac{q^2Y}{r^5} - A_{322}\frac{qX}{r^4},$$

$$P_Z = (A_{121}+A_{211})\frac{qZ^2}{r^5} + (A_{221}+A_{212}+A_{122})\frac{q^2Z}{r^5} + A_{111}\frac{Z^3}{r^5} + A_{222}\frac{q^3}{r^5}.$$

4. INTRODUCTION OF POSTULATE III

According to Postulate III, the force dF_Z, produced by any closed circuit made up of elements $I_1 d\mathbf{s}_1$, is zero. Thus,

$$dF_Z = \oint (P_X dX + P_Y dY + P_Z dZ)$$
$$= \oint \mathbf{P} \cdot d\mathbf{s}_1 = 0, \tag{8}$$

where

$$\mathbf{P} = \mathbf{i} P_X + \mathbf{j} P_Y + \mathbf{k} P_Z,$$
$$d\mathbf{s}_1 = \mathbf{i}\, dX + \mathbf{j}\, dY + \mathbf{k}\, dZ.$$

By Green's theorem,

$$\oint_C \mathbf{P} \cdot d\mathbf{s}_1 = \int_S \operatorname{curl} \mathbf{P} \cdot d\mathbf{A} = 0, \tag{9}$$

where S is any surface bounded by the closed circuit C. Since Eq. 9 must hold for all surfaces S,

$$\operatorname{curl} \mathbf{P} \equiv 0, \tag{10}$$

or

$$\frac{\partial P_X}{\partial Y} = \frac{\partial P_Y}{\partial X}, \quad \frac{\partial P_X}{\partial Z} = \frac{\partial P_Z}{\partial X}, \quad \frac{\partial P_Y}{\partial Z} = \frac{\partial P_Z}{\partial Y}.$$

Substitution of Eq. 7 into Eq. 10 yields the following conditions which must be satisfied to meet Postulate III:

$$\begin{cases} A_{121} = A_{211} = A_{222} = A_{322} = 0, \\ A_{312} = -A_{321}, \\ A_{212} = 2A_{122} - 2A_{111} - A_{221}. \end{cases} \tag{11}$$

Thus the most general equations that satisfy the four postulates are

$$\begin{cases} d^2F_r = \dfrac{|I_1||I_2|ds_1 ds_2}{r^2} [A_{111} \cos\theta_1 \cos\theta_2 + A_{221} \sin\theta_1 \sin\theta_2 \cos\eta \\ \qquad + A_{321} \sin\theta_1 \sin\theta_2 \sin\eta], \\ d^2F_n = \dfrac{|I_1||I_2|ds_1 ds_2}{r^2} [A_{122} \cos\theta_1 \sin\theta_2 \\ \qquad + (2A_{122} - 2A_{111} - A_{221}) \sin\theta_1 \cos\theta_2 \cos\eta \\ \qquad - A_{321} \sin\theta_1 \cos\theta_2 \sin\eta], \\ d^2F_p = \dfrac{|I_1||I_2|ds_1 ds_2}{r^2} [-A_{321} \sin\theta_1 \cos\theta_2 \cos\eta \\ \qquad + (2A_{122} - 2A_{111} - A_{221}) \sin\theta_1 \cos\theta_2 \sin\eta \\ \qquad + A_{123} \cos\theta_1 \sin\theta_2 + A_{223} \sin\theta_1 \sin\theta_2 \cos\eta \\ \qquad + A_{323} \sin\theta_1 \sin\theta_2 \sin\eta], \\ \qquad i, j = 1, 2, 3, \end{cases} \tag{12}$$

where the forces are along the three coordinate axes of Fig. 1.

5. PARALLEL CONDUCTORS

We have now employed all four postulates and by this means have reduced the original 18 constants of Eq. 3 to the 7 constants of Eq. 12. Still further reduction can be effected by using the well-known equation for force between parallel conductors:

$$dF_X = 0, \qquad dF_Y = \frac{\mu |I_1||I_2| ds_2}{2\pi l}. \tag{13}$$

The subscripts refer to the axes of Fig. 2. Equation 13 expresses the force on the element $I_2 ds_2$, produced by an infinitely long wire, Fig. 3.

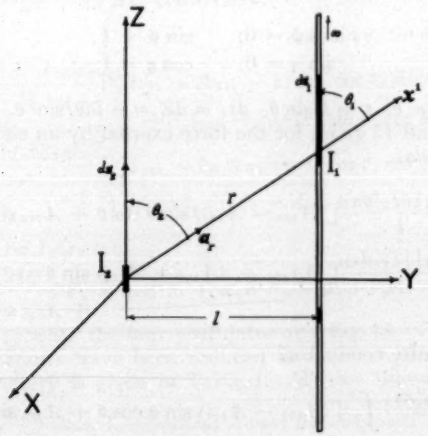

FIG. 3.

From the geometry of Fig. 2,

$$\begin{cases} d^2F_X = d^2F_r \sin\theta_2 \cos\psi - d^2F_n \cos\theta_2 \cos\psi + d^2F_p \sin\psi, \\ d^2F_Y = d^2F_r \sin\theta_2 \sin\psi - d^2F_n \cos\theta_2 \sin\psi + d^2F_p \cos\psi. \end{cases} \tag{13a}$$

Substitution of Eq. 12 yields

$$\begin{aligned} d^2F_X = \frac{|I_1||I_2| ds_1 ds_2}{r^2} \big[& (A_{111} - A_{122}) \cos\theta_1 \sin\theta_2 \cos\theta_2 \cos\psi \\ & + A_{221} \sin\theta_1 \sin^2\theta_2 \cos\eta \cos\psi + A_{321} \sin\theta_1 \sin\eta \cos\psi \\ & - (2A_{122} - 2A_{111} - A_{221}) \sin\theta_1 \cos^2\theta_2 \cos\eta \cos\psi \\ & - A_{321} \sin\theta_1 \cos\theta_2 \cos\eta \sin\psi \\ & + (2A_{122} - 2A_{111} - A_{221}) \sin\theta_1 \cos\theta_2 \sin\eta \sin\psi \\ & + A_{123} \cos\theta_1 \sin\theta_2 \sin\psi + A_{223} \sin\theta_1 \sin\theta_2 \cos\eta \sin\psi \\ & + A_{323} \sin\theta_1 \sin\theta_2 \sin\eta \sin\psi \big], \end{aligned} \tag{14}$$

$$d^2F_Y = \frac{|I_1||I_2|ds_1ds_2}{r^2}[(A_{111} - A_{122})\cos\theta_1\sin\theta_2\cos\theta_2\sin\psi$$
$$+ A_{221}\sin\theta_1\sin^2\theta_2\cos\eta\sin\psi + A_{321}\sin\theta_1\sin\eta\sin\psi$$
$$- (2A_{122} - 2A_{111} - A_{221})\sin\theta_1\cos^2\theta_2\cos\eta\sin\psi$$
$$+ A_{321}\sin\theta_1\cos\theta_2\cos\eta\cos\psi$$
$$- (2A_{122} - 2A_{111} - A_{221})\sin\theta_1\cos\theta_2\sin\eta\cos\psi$$
$$- A_{123}\cos\theta_1\sin\theta_2\cos\psi - A_{223}\sin\theta_1\sin\theta_2\cos\eta\cos\psi$$
$$- A_{323}\sin\theta_1\sin\theta_2\sin\eta\cos\psi]. \quad (15)$$

For any element in the YZ-plane,

$$\cos\psi = 0, \qquad \sin\psi = 1;$$
$$\sin\eta = 0, \qquad \cos\eta = 1.$$

Also, $\theta_1 = \theta_2 = \theta$, $r = l/\sin\theta$, $ds_1 = dZ = -ld\theta/\sin^2\theta$. Substitution into Eqs. 14 and 15 gives, for the force exerted by an element I_1ds_1 on the element I_2ds_2,

$$\begin{cases} d^2F_X = -\dfrac{|I_1||I_2|ds_2}{l}[(A_{123} - A_{321})\sin\theta\cos\theta + A_{223}\sin^2\theta]d\theta, \\ d^2F_Y = -\dfrac{|I_1||I_2|ds_2}{l}[(3A_{111} - 3A_{122} + A_{221})\sin\theta\cos^2\theta \\ \qquad\qquad + A_{221}\sin^3\theta]d\theta. \end{cases} \quad (16)$$

For an infinite conductor,

$$dF_X = \frac{|I_1||I_2|ds_2}{l}\int_0^\pi [(A_{123} - A_{321})\sin\theta\cos\theta + A_{223}\sin^2\theta]d\theta$$
$$= \frac{|I_1||I_2|ds_2}{l}\frac{\pi}{2}A_{223}. \quad (17)$$

But $dF_X = 0$, so $A_{223} = 0$.

Similarly,

$$dF_Y = \frac{|I_1||I_2|ds_2}{l}\int_0^\pi [(3A_{111} - 3A_{122} + A_{221})\sin\theta\cos^2\theta$$
$$+ A_{221}\sin^3\theta]d\theta$$
$$= 2\frac{|I_1||I_2|ds_2}{l}(A_{111} - A_{122} + A_{221}). \quad (18)$$

Employing Eq. 13, we obtain the relation,

$$A_{122} = A_{111} + A_{221} - \mu/4\pi. \quad (19)$$

6. GENERAL RESULTS

A study was made of all the special cases obtainable by taking one coefficient at a time and equating the other coefficients to zero. If all A_{ijk} are zero except A_{321}, two different forces are predicted for two cases that are geometrically identical. The only way of achieving consistency is to set A_{321} equal to zero.

The most general equations for the force between current elements are now obtained from Eq. 12 by incorporating the above datum and the results of Section 5:

$$\begin{cases} d^2F_r = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}\,[A_{111}\cos\theta_1\cos\theta_2 \\ \qquad\qquad + A_{221}\sin\theta_1\sin\theta_2\cos\eta], \\ d^2F_n = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}\,[(A_{111} + A_{221} - \mu/4\pi)\cos\theta_1\sin\theta_2 \\ \qquad\qquad + (A_{221} - 2\mu/4\pi)\sin\theta_1\cos\theta_2\cos\eta], \\ d^2F_p = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}\,[(A_{221} - 2\mu/4\pi)\sin\theta_1\cos\theta_2\sin\eta \\ \qquad\qquad + A_{123}\cos\theta_1\sin\theta_2 + A_{323}\sin\theta_1\sin\theta_2\sin\eta]. \end{cases} \tag{20}$$

The force on I_2ds_2 is

$$d^2\mathbf{F}_2 = \mathbf{a}_r\,d^2F_r + \mathbf{a}_n\,d^2F_n + \mathbf{a}_p\,d^2F_p. \tag{21}$$

Equation 20 satisfies the four postulates and Eq. 13. The original 18 arbitrary constants have been reduced to four: A_{111}, A_{221}, A_{123}, and A_{323}. A summary is given in Table I. We see that *the Ampère ex-*

TABLE I.—*Relations Among the Constants.*

Symmetry relations

$$\begin{cases} A_{112} = A_{113} = 0, \\ A_{211} = A_{211}, \\ A_{313} = A_{212}, \\ A_{213} = A_{312}. \end{cases}$$

Postulate III

$$\begin{cases} A_{121} = A_{211} = A_{222} = A_{322} = 0, \\ A_{312} = -A_{321}, \\ A_{212} = 2A_{122} - 2A_{111} - A_{221}. \end{cases}$$

Parallel conductors

$$\begin{cases} A_{223} = 0, \\ A_{122} = A_{111} + A_{221} - \mu/4\pi. \end{cases}$$

Consistency

$$A_{321} = 0.$$

The above relations reduce the 18 original coefficients A_{ijk} to four: A_{111}, A_{221}, A_{123}, A_{323}.

periments are by no means sufficient to uniquely determine an equation for the force between current elements. Indeed, Eq. 20 represents ∞^4 possible solutions of the problem.

Equations 20 and 21 express the force in terms of components in the directions of $\mathbf{a}_r$, $\mathbf{a}_\eta$, and $\mathbf{a}_\rho$ (Fig. 1). In terms of the coordinate system of Fig. 2, the force is

$$d^2\mathbf{F}_2 = \mathbf{i}\, d^2F_X + \mathbf{j}\, d^2F_Y + \mathbf{k}\, d^2F_Z, \tag{22}$$

where

$$\begin{aligned} d^2F_X = \frac{|I_1||I_2|ds_1ds_2}{r^2}\,[&-(A_{221}-\mu/4\pi)\cos\theta_1\sin\theta_2\cos\theta_2\cos\psi \\ &+ A_{221}\sin\theta_1\sin^2\theta_2\cos\eta\cos\psi \\ &+ (A_{221}-2\mu/4\pi)(\sin\eta\sin\psi-\cos\theta_2\cos\eta\cos\psi)\sin\theta_1\cos\theta_2 \\ &+ A_{123}\cos\theta_1\sin\theta_2\sin\psi + A_{223}\sin\theta_1\sin\theta_2\sin\eta\sin\psi], \end{aligned} \tag{14a}$$

$$\begin{aligned} d^2F_Y = \frac{|I_1||I_2|ds_1ds_2}{r^2}\,[&-(A_{221}-\mu/4\pi)\cos\theta_1\sin\theta_2\cos\theta_2\sin\psi \\ &+ A_{221}\sin\theta_1\sin^2\theta_2\cos\eta\sin\psi \\ &- (A_{221}-2\mu/4\pi)(\sin\eta\cos\psi+\cos\theta_2\cos\eta\sin\psi)\sin\theta_1\cos\theta_2 \\ &- A_{123}\cos\theta_1\sin\theta_2\cos\psi - A_{223}\sin\theta_1\sin\theta_2\sin\eta\cos\psi], \end{aligned} \tag{15a}$$

$$\begin{aligned} d^2F_Z = \frac{|I_1||I_2|ds_1ds_2}{r^2}\,[&A_{111}\cos\theta_1 + (A_{221}-\mu/4\pi) \\ &\times(2\sin\theta_1\cos\theta_2\cos\eta+\cos\theta_1\sin\theta_2)\sin\theta_2]. \end{aligned} \tag{6a}$$

7. SPECIAL CASES

Now consider some special cases of Eq. 20.

A. The force is in the direction of $\mathbf{a}_r$.

Then $d^2F_\eta = d^2F_\rho = 0$, and each coefficient in the second and third equations of (20) must be zero:

$$\begin{cases} A_{111} + A_{221} - \mu/4\pi = 0, \\ A_{221} - 2\mu/4\pi = 0, \\ A_{123} = A_{223} = 0, \end{cases}$$

or $A_{111} = -\mu/4\pi$, $A_{221} = 2\mu/4\pi$.

Equation 21 reduces to

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\,[2\sin\theta_1\sin\theta_2\cos\eta - \cos\theta_1\cos\theta_2], \tag{1}$$

which is *Ampère's equation.* Thus we have shown that Ampère's equation is a degenerate form of Eq. 20 and that it is the only equation

that satisfies the four postulates, Eq. 13, and the requirement that the force be along the line of centers.

As Grassmann (4) pointed out, however, the Ampère expression behaves in a peculiar manner. For instance, if $\eta = 0$ and $\theta_1 = \theta_2 = \theta$, the force becomes zero when

$$2 \sin^2 \theta = \cos^2 \theta$$

or when

$$\tan \theta = 1/\sqrt{2}, \qquad \theta = 35.3°.$$

For angles less than 35.3°, the force is negative; for angles between 35.3° and 90°, the force is positive. Though one cannot say that such behavior is impossible, it does seem a very unlikely way for nature to operate.

B. The force is in the direction of $\mathbf{a}_n$.

The equations are inconsistent, proving that such a case is impossible. The same conclusion applies to $d^2\mathbf{F}_2$ along $\mathbf{a}_p$.

C. The force is perpendicular to $\mathbf{a}_r$.

Then $d^2F_r = 0$, and therefore $A_{111} = A_{221} = 0$. Equation 20 reduces to

$$\begin{cases} d^2F_n = -\dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}[\cos\theta_1 \sin\theta_2 + 2\sin\theta_1 \cos\theta_2 \cos\eta], \\ d^2F_p = -\dfrac{2\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\sin\theta_1 \cos\theta_2 \sin\eta \\ \quad + \dfrac{|I_1||I_2|ds_1ds_2}{r^2}[A_{123}\cos\theta_1 \sin\theta_2 + A_{323}\sin\theta_1 \sin\theta_2 \sin\eta]. \end{cases} \tag{20a}$$

If, in addition, we set $A_{123} = A_{323} = 0$, we obtain another fairly simple expression which satisfies the Ampère requirements.

D. The force is perpendicular to $\mathbf{a}_n$.

Then $d^2F_n = 0$ and $A_{221} = 2\mu/4\pi$, $A_{111} = -\mu/4\pi$. Thus

$$\begin{cases} d^2F_r = \dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}[2\sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2], \\ d^2F_p = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}[A_{123}\cos\theta_1 \sin\theta_2 \\ \qquad\qquad + A_{323}\sin\theta_1 \sin\theta_2 \sin\eta]. \end{cases} \tag{20b}$$

If $A_{123} = A_{323} = 0$, we obtain the Ampère equation. Other choices of these constants yield a variety of other equations, all satisfying the Ampère requirements.

E. The force is perpendicular to $\mathbf{a}_p$.

Then $d^2F_p = 0$ and $A_{122} = A_{322} = 0$, $A_{221} = 2\mu/4\pi$, and

$$\begin{cases} d^2F_r = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}\left[A_{111}\cos\theta_1\cos\theta_2 + \dfrac{2\mu}{4\pi}\sin\theta_1\sin\theta_2\cos\eta\right], \\ d^2F_n = \dfrac{|I_1||I_2|ds_1ds_2}{r^2}\left|(A_{111} + \mu/4\pi)\cos\theta_1\sin\theta_2\right|. \end{cases} \tag{20c}$$

If we let $A_{111} = -\mu/4\pi$, the second equation disappears and we obtain again Ampère's equation.

But if $A_{111} = 0$,

$$\begin{cases} d^2F_r = \dfrac{2\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\sin\theta_1\sin\theta_2\cos\eta, \\ d^2F_n = \dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\cos\theta_1\sin\theta_2. \end{cases} \tag{20d}$$

F. The force is perpendicular to the element ds_2.

Then $d^2F_z = 0$, and from Eq. 6*a*

$$A_{111} = 0, \qquad A_{221} = \mu/4\pi.$$

Thus Eq. 20 becomes

$$\begin{cases} d^2F_r = \dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\sin\theta_1\sin\theta_2\cos\eta, \\ d^2F_n = -\dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\sin\theta_1\cos\theta_2\cos\eta, \\ d^2F_p = -\dfrac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2}\sin\theta_1\cos\theta_2\sin\eta + \dfrac{|I_1||I_2|ds_1ds_2}{r^2} \\ \qquad \times\left[A_{122}\cos\theta_1\sin\theta_2 + A_{322}\sin\theta_1\sin\theta_2\sin\eta\right]. \end{cases} \tag{20e}$$

If we let $A_{122} = A_{322} = 0$, Eq. 20*e* reduces to Grassmann's equation (2*a*).

8. OTHER POSTULATES

The foregoing results are all based on the four postulates inferred from Ampère's experiments. But none of the equations is completely satisfactory. It is true that current elements are a fiction, so that physical intuition must not be relied upon too strongly in determining allowable forms of the force equation for current elements. Yet a current element can be considered as a complex of moving charges, for

which we prefer to keep Newton's third law. And this law is violated (16) by the Grassmann equation (2) and by all of the foregoing equations except Ampère's.

If action is to be equal to reaction, therefore, Eq. 1 is the only possibility consistent with the four postulates. But Eq. 1 has the peculiarity pointed out by Grassmann, that the force changes sign at $\theta = 35.3°$. Is it conceivable that one of the four postulates is incorrect?

Examination of the postulates (stated in Section 2) shows that I, II, and IV can hardly be questioned. Certainly the force should vary directly with the current, directly with the length of element, and inversely as the square of the distance. Also, one should be able to replace ds by its components. But Postulate III is in a much less secure position. All four of the Ampère experiments dealt with complete circuits. Experiment 3 differed from the others, however, in employing a short conductor capable of movement with respect to the rest of its circuit. This movable portion was supposed to represent a current element; and the fact that it experienced no longitudinal force was supposed to prove that a current element can experience no longitudinal force. But a portion of a circuit is not identical with an isolated current element having no leads; and the experimental result may be caused by the balancing of forces which would not balance in a current element.

Suppose that Postulate III be replaced by Ampère's fifth postulate:

Postulate V. The force between current elements acts along the line of centers, and action is equal to reaction.

Then $A_{ij2} = A_{ij3} = 0$. From Eqs. 3 and 5,

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{|I_1||I_2|ds_1ds_2}{r^2}[A_{111}\cos\theta_1\cos\theta_2 + A_{211}\sin\theta_1\cos\theta_2(\sin\eta + \cos\eta) + A_{121}\cos\theta_1\sin\theta_2 + A_{221}\sin\theta_1\sin\theta_2\cos\eta + A_{321}\sin\theta_1\sin\theta_2\sin\eta]. \quad (23)$$

If action and reaction are equal,

$$A_{121} = -A_{211}, \quad (24)$$

leaving four independent constants: A_{111}, A_{211}, A_{221}, and A_{321}. Now consider the force produced by parallel conductors. As in Section 5, the force on element I_2ds_2, caused by current I_1 in an infinite conductor (Fig. 3), is

$$dF_Y = \frac{\mu|I_1||I_2|ds_2}{2\pi l}. \quad (13)$$

From Eq. 13a, for $\psi = \pi/2$ and $\eta = 0$,

$$d^2F_Y = \sin\theta_2 \cdot d^2F_r, \quad (25)$$

and, for $\theta_1 = \theta_2 = \theta$,

$$d^2F_r = \frac{|I_1||I_2|ds_1ds_2}{r^2}[A_{111}\cos^2\theta + A_{221}\sin^2\theta]$$

$$= -\frac{|I_1||I_2|ds_2}{l}[A_{111}\cos^2\theta + A_{221}\sin^2\theta]d\theta. \quad (26)$$

Substitution of Eq. 26 into Eq. 25 gives

$$dF_Y = \frac{|I_1||I_2|ds_2}{l}\int_0^\pi [A_{111}\sin\theta\cos^2\theta + A_{221}\sin^3\theta]d\theta$$

$$= \frac{2}{3}\left(\frac{|I_1||I_2|ds_2}{l}\right)(A_{111} + 2A_{221}). \quad (27)$$

By use of Eq. 13, we obtain

$$A_{221} = \frac{3\mu}{8\pi} - A_{111}/2. \quad (28)$$

Thus *the most general equation for force* on a current element (consistent with Postulates I, II, IV, and V but not III) must be

$$d^2\mathbf{F}_2 = \mathbf{a}_r\frac{|I_1||I_2|ds_1ds_2}{r^2}\Big\{A_{111}\cos\theta_1\cos\theta_2$$

$$+\left(\frac{3\mu}{8\pi} - A_{111}/2\right)\sin\theta_1\sin\theta_2\cos\eta$$

$$+A_{211}[\sin\theta_1\cos\theta_2(\sin\eta + \cos\eta) - \cos\theta_1\sin\theta_2]$$

$$+A_{321}\sin\theta_1\sin\theta_2\sin\eta\Big\}. \quad (29)$$

A number of special cases, derived from Eq. 29, look promising. Consider equations having not more than two terms.

1. Suppose $A_{321} \neq 0$, but $A_{211} = 0$. Then there are only two degenerate cases:

$$(a)\ A_{111} = 0,$$

$$d^2\mathbf{F}_2 = \mathbf{a}_r\frac{|I_1||I_2|ds_1ds_2}{r^2}\left[\frac{3\mu}{8\pi}\cos\eta + A_{321}\sin\eta\right]\sin\theta_1\sin\theta_2. \quad (29a)$$

$$(b)\ A_{111} = 3\mu/4\pi,$$

$$d^2\mathbf{F}_2 = \mathbf{a}_r\frac{|I_1||I_2|ds_1ds_2}{r^2}$$

$$\times\left[\frac{3\mu}{4\pi}\cos\theta_1\cos\theta_2 + A_{321}\sin\theta_1\sin\theta_2\sin\eta\right]. \quad (29b)$$

2. Suppose $A_{221} = 0$ and $A_{221} = 0$. Then there are three simple cases:

$$(a)\quad A_{111} = 0,$$

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{3\mu|I_1||I_2|ds_1ds_2}{8\pi r^2} \sin\theta_1 \sin\theta_2 \cos\eta. \tag{29c}$$

$$(b)\quad A_{111} = 3\mu/4\pi,$$

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{3\mu|I_1||I_2|ds_1ds_2}{4\pi r^2} \cos\theta_1 \cos\theta_2. \tag{29d}$$

$$(c)\quad A_{111} = -\mu/4\pi,$$

$$d^2\mathbf{F}_2 = \mathbf{a}_r \frac{\mu|I_1||I_2|ds_1ds_2}{4\pi r^2} [2 \sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2]. \tag{1}$$

The last equation is Ampère's formula.

Equations 29*c* and 29*d* are simpler than Ampère's equation and deserve further study, particularly since they are free from the peculiarity of Ampère's formula mentioned by Grassmann. Of the two, Eq. 29*c* appears to be the better. Equation 29*d* predicts that two perpendicular elements ($\theta_1 = \theta_2 = \pi/2$, $\eta = \pi/2$) will react exactly as if they were in the same plane ($\eta = 0$), which seems unlikely. It also requires that parallel elements exert zero force on each other, which is at least contrary to popular belief. Equation 29*c*, on the other hand, predicts zero force for collinear elements, zero force if $\eta = \pi/2$, and maximum force for parallel elements that are perpendicular to $\mathbf{a}_r$. These predictions seem reasonable.

9. CONCLUSIONS

The paper has developed the most general equations obtainable on two bases; the first including Postulate III but not V, the second including V but not III. The former leads to the general equation (20) with four arbitrary constants; the latter to Eq. 29 with three arbitrary constants.

Special cases of Eq. 20 include Ampère's equation (1), Grassmann's equation (2), and a host of others. The only one that agrees with Newton's third law, however, is Ampère's original equation; and this equation has the weakness that the force changes sign in a peculiar manner.

Special cases of Eq. 29 include Ampère's equation and a simpler form, Eq. 29*c*, which is free from the weakness of Ampère's equation. Further work will be necessary to determine which of the many possible equations is most satisfactory.

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RAPID ESTIMATION METHOD FOR LONG-POINTED PROJECTILE TRAJECTORIES

BY

HOMER S. POWLEY[1]

INTRODUCTION

The determination of projectile trajectories at practical velocities has been exceedingly difficult, complex, laborious and costly. After the basic differential equations of motion are set up, producing what Cranz (1)[2] refers to as the Chief Equation, the trajectory can be determined by integration. To quote Cranz (1) however, "strictly speaking, the problem cannot be solved analytically in a finite form." In general, one has recourse to either (*a*) being content with an approximate solution or (*b*) starting with an approximation of the Chief Equation and obtaining a simplified solution. In other words, an approximate solution is all that can be expected to be obtained.

Fortunately, however, methods of numerical integration are possible which give approximations with as high a degree of refinement as may be required, depending entirely upon the amount of labor and expense employed. This applies to (*a*) above. Formerly, large computing staffs were employed and the length of time was great. In recent years, quite expensive electronic numerical integrating machines give solutions of high accuracy in relatively short times.

With situation (*b*) above, the Siacci Method, which is the one most generally used, gives entirely satisfactory practical results provided that the method is limited to the determination of short, flat, trajectories. See Bliss (2, p. 37, ff.) for the method of Hitchcock and Kent. The amount of work necessary to prepare tables for the Siacci Method is considerable and the work for determination of air resistance which forms the basis of the table is even more considerable and costly.

Many problems in shooting would be greatly simplified if one or two points could be determined on a trajectory by experimental shooting and then the rest of the trajectory became known. Alternately, if one or two points on a trajectory are determined by theory, the rest of the trajectory becomes available in simple fashion and practical benefits would be appreciable. The present paper shows one means to this end.

DISCUSSION

The Western Cartridge Company (3) publishes trajectory curves for military cartridges based on the statistical results of actual shooting

[1] Consultant, Shaker Heights, Ohio. Formerly, Plant Manager, Coldstream Products Company, Denison, Texas.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

with said cartridges. The author noticed that when data were plotted on logarithmic paper, practically a straight line resulted. The immediate possibility became apparent that a trajectory curve might be considered, from a practical standpoint, as that from a simple exponential equation. The author, being a practical rifle shooter, fired some trajectories which further substantiated this supposition. Further, examination of other published rifle trajectories served to confirm the belief.

Table I is extracted from p. 294 of Johnson and Haven (4) with the drop in feet being converted from the values given.

TABLE I.

Caliber .50 Ball, 2700 f/s.

Range, Yd.	Elevation, Mils	Drop in Feet
100	0.8	0.24
200	1.6	0.94
300	2.4	2.12
400	3.2	3.78
500	4.0	5.9
600	4.9	8.6
700	5.8	12.0
800	6.8	16.0
900	7.9	21.3
1000	9.1	26.8
1100	10.4	33.7
1200	11.8	41.8
1300	13.3	50.9

The data from Table I are plotted in Fig. 1 with the straight line being drawn with an angle to the horizontal whose tangent equals 2.3. The reason for the selection of the value 2.3 will become evident later. Nevertheless, the approximation of the plot of the data to a straight line is quite evident. The data in the table were determined by the U. S. Army and the author does not know by what methods. The projectile is long-pointed and boat-tailed.

Becker and Cranz (5) published Drag determinations made with a Spitzgeschoss (pointed shot) projectile, or Spitzer bullet. Analysis showed that, for velocities above 1200 f/s, the data showed for a projectile weighing 1 lb. and having a diameter of 1 in.,

$$\text{Drag} = 129.0 - 0.221\ V$$

where V is the velocity in f/s. The numerical values were determined from the Drag data, after conversion from the metric, by the Method of Least Squares.

This provides a starting point for obtaining the well-known Siacci

Space and Time functions by direct integration of the linear equation for the Drag. The solutions follow.

$$S = 4.525\ (V_0 - V) + 583.7\ T$$

$$T = 10.42 \log_{10} \frac{V_0 - 583.7}{V - 583.7}$$

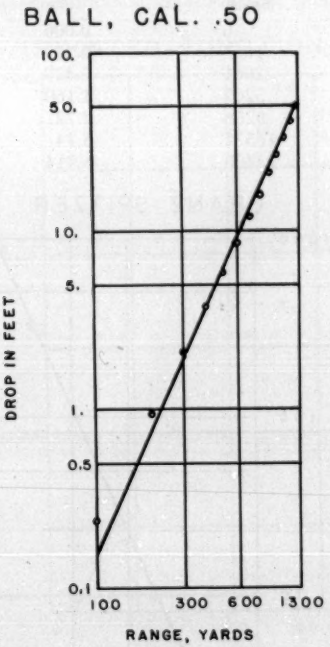

FIG. 1.

where S is the space function, V_0 is the initial velocity in f/s, V is the terminal velocity in f/s and T is the time of flight in seconds.

The Remington Arms Company has recently publicized a drop formula originated by their Dr. C. S. Cummings (6). This applies to projectiles fired in a substantially horizontal direction and follows.

$$\text{Drop} = \tfrac{1}{2}gt^2 \left(1 - \frac{Bt}{3} + \frac{B^2t^2}{12}\right)$$

where $B = (V_0 - V)/x$, x = range in feet, and t = time in seconds.

Table II has been calculated for an initial velocity of 5000 f/s downward showing the Space and Time functions as well as the drop in feet from a horizontal line of departure by the equations just given. Data from Table II are plotted in Fig. 2. The tangent of angle of slope

TABLE II.

Velocity	Space	Time	Drop in Feet
5000	0	0.000	0.0
4800	1027	0.209	0.7
4600	2061	0.429	2.9
4000	5202	1.160	20.2
3000	9208	2.72	98.4
2000	16577	5.14	324.
1200	22400	8.914	878.

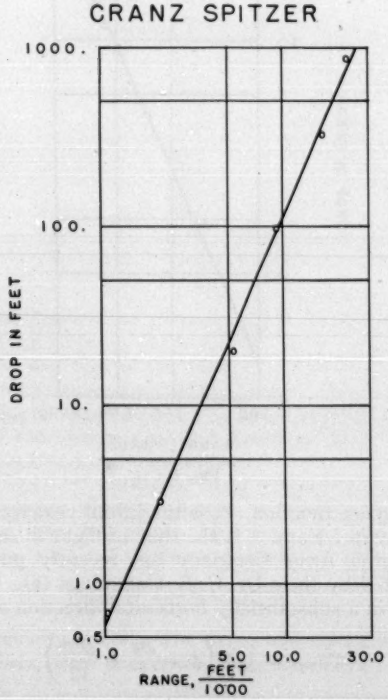

FIG. 2.

of the straight line was determined by the Method of Least Squares to be 2.3, very closely. It is realized that we are dealing with two sets of approximations. The first approximation is that the air resistance measurements show the Drag to be a linear function of the velocity. The Cranz data are indeed a close approximation. The second approximation is contained in the Cummings formula. The author has applied this formula to rather a large amount of theoretical and experi-

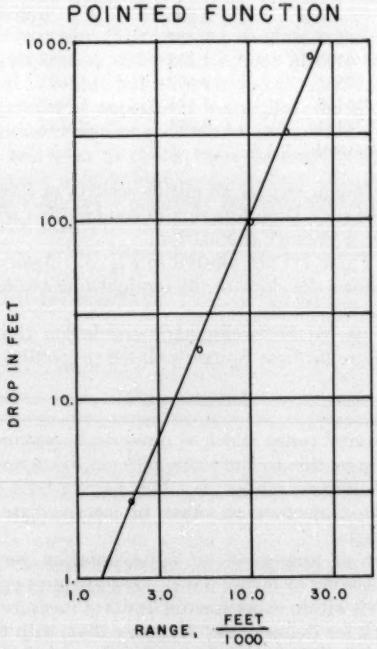

FIG. 3.

mental rifle data and believes it to be correct within practical limits of measurement at least while the projectile is traveling at supersonic velocities.

The author has compared the small amounts of published Drag data on pointed flat-base projectiles. Among these are the Hodsock (7). From such considerations, he decided to use the following equation:

$$\text{Drag} = 146.6 - 0.228\ V.$$

He then prepared on this somewhat arbitrary basis a complete Siacci Table from 5000 f/s downward to 1200 f/s. This table, not given here, shows Space, Time, Altitude and Inclination Functions. Based on these functions the drop in feet from a horizontal firing has been calculated. Table III is an abstract of the Siacci-type table with the

TABLE III.

Velocity	Space	Time	Altitude	Drop in Feet
5000	0.0	0.000	0.00	0.0
4600	2025.45	0.422	2030.67	2.6
4000	5120.06	1.143	5158.15	19.
3000	10501.	2.697	10697.	98.
2000	16446.	5.115	17067.	311.
1200	22451.	9.044	24133.	841.

Inclination at 5000 f/s = 1.00000.

addition of the drop in feet for an initial velocity of 5000 f/s. Thus we have used an arbitrary Drag function in combination with an accepted, sound method of trajectory calculation.

The data of Table III are plotted in Fig. 3. Again, the Method of Least Squares shows the slope of the straight line assumed to represent the drop as 2.3, very closely.

This brings us to the preliminary conclusion that the following equation applies to flat-base Spitzer-pointed projectiles:

$$\text{Drop} = kx^{2.3}$$

where x = horizontal range and k = numerical constant. For as good an approximation as this is, the value of k can be determined by firing for drop at two different ranges, x. This having been determined, the remainder of the trajectory at other or intermediate points can be simply determined.

The question of how good an approximation we have is to be determined by existing or future data. The author's comparisons have shown it to be well within experimental limits of measurements. Others will wish to check for themselves. Suppose that, with the U. S. service rifle the method indicates the drop at 500 yards to be, say, 76 in. Under ideal air conditions, the expected dispersion of five or ten shots cannot be expected to be less than 1 minute of angle. This would be 5 in. at 500 yards, approximately, something over 6 per cent total spread or plus or minus 3 per cent in observed drop. All of the data thus far examined seemed to be well within this range by formula.

Assuming excellent experimental agreement with the equation, one might like to think that such an exponential equation might implicitly describe trajectories. It does not. Mathematically, it is possible to start with a known trajectory, determined with sufficient accuracy, and

arrive at other ballistic elements such as time, velocity, and Drag. (See Cranz (1), p. 93.) In order to test this, a trajectory was determined by starting with known initial and final velocities, times, distances and Drag. Then, using the exponential equation as a starting point, a terminal velocity was determined. This turned out to be at least 10 per cent in error. Likewise, a time of flight was attempted to be re-created and was found to be in error about 6 per cent. Further, the Drag was attempted to be re-created and found to be more than 20 per cent in error.

Thus, trajectories, in their precise qualities, remain costly and laborious of determination and we have been able to find an equation which is empirical in nature but nevertheless of practical use only while the projectile remains at supersonic velocities. It is, however, quite useful for all manner of sporting rifle uses and airplane cannon.

Since others will wish to check these findings, it is worthwhile to mention that similar calculations indicate that horizontal deflections of the trajectories due to a constant ballistic side wind are directly proportional to the 2.15 power of the range so long as velocities are upwards of 1400 f/s. Also, blunt-nose projectiles, such as the Krupp, do not follow an exponential curve for drop. Boat-tail projectiles have not been fully investigated.

CONCLUSION

It has been discovered that, within practical limits of measurement for projectiles with long-pointed heads at supersonic velocities, the downward curve of the trajectory in air from a substantially horizontal firing is directly proportional to the 2.3 power of the range. This represents a very great simplification of trajectory determinations and permits of very rapid and accurate estimations with a minimum of data.

It is hoped that additional determinations by others will extend the value of this work. This would immediately show the way to simplification of the machinery necessary for the control of automatic firing mechanisms of various types.

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Aerosol Spray Softens Glare in Photographing Shiny Objects.—A glare-softening coating material designed to help the commercial photographer or TV cameraman present better pictures of bright and shiny objects now is available in easy-to-use, self-spraying aerosol containers. It is a product of the Crescent Portrait and Frame Company, Cleveland, Ohio.

Formulated from the same acrylic materials used to manufacture many of the modern plastics, the compound was developed primarily for giving glossy photo prints a matte surface which would eliminate glaring highlights, or "hot spots," that occurred by reflection under TV camera lights. It is equally effective, according to its manufacturer, on any shiny surface such as cellophane bags, chrome furniture, glass picture frames, costume jewelry, or glassware.

In the photographic field, the spray product is expected to provide a solution to the problem of copying high-gloss color prints. Tests conducted by a manufacturer of photographic color print material indicate the glare-cutting compound has no adverse effects on the dyes in its color prints ("Printon" color prints, Ansco Division of General Aniline & Film Corporation).

The spray-it-on solution, its manufacturer says, also has the added advantage of covering small marks, scratches, and blemishes on photo prints. It provides an excellent tooth for retouching or spotting. Transparent oil colors can be applied over the coating after it dries for about 30 minutes. In an emergency, the manufacturer adds, the solution can be sprayed on ordinary window glass to obtain a ground glass effect.

Best results are obtained when the solution is sprayed from a distance of 6 to 10 in. and at an angle of approximately 20 deg. from the plane of the surface. The finest textured surface is produced when the spray is used at about 90° F.

The acrylic base product is insoluble in water but can be removed easily with a companion product containing toluene as a solvent. Its manufacturer also expects to introduce shortly a nonflammable, water-soluble, anti-glare solution for temporary treatment of shiny objects. It, too, will be packaged in an aerosol container of the type which uses Du Pont "Freon" fluorinated hydrocarbon as the propellent.

Vibration Machine to Explore Man's Reactions to Jiggling.—A machine has been developed by researchers at the Naval Medical Center, Bethesda, Md., to explore the human body's reactions to constant vibration. Called a "Large Displacement-Amplitude Vibration Machine," the new device consists of a platform actuated by a 25-hp. motor, which can move a man up and down 50 times a second and (at lower frequencies) bounce him through a distance of 4 in. The machine is designed for a maximum load of 200 lb. at any combination of displacements and frequencies not exceeding 15-g. peak acceleration. It is hoped that it will enable designers to set vibration tolerances in design of new equipment.

Giant Sphere for Atomic Sub Engine Completed.—The Atomic Energy Commission announced recently that the giant steel sphere at West Mihon, N. Y., which will be used for testing experimental submarine atomic power plants, has been completed. The big globe is 225 ft. in diameter and has 5,400,000 cu. ft. of free space. Its thick skin, composed of 682-in. thick, six-ton steel plates welded together, will shield the surrounding population in the remote event that all the reactor's built-in safety controls fail simultaneously.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

NBS PARACHUTE TELEMETERING SYSTEM

A parachute telemetering system recently developed by the National Bureau of Standards is facilitating tests of experimental parachutes for use with modern high-speed aircraft. Electronic equipment mounted inside a parachute-borne torso-shaped dummy transmits by radio needed information—altitude, and forces at various points—in coded form to a ground station, where the information is decoded. The system was developed for the Navy Bureau of Aeronautics by M. L. Greenough, C. C. Gordon and associates of NBS.

Military parachutes have been required in increasing numbers in recent years as military aviation has continued to advance. Parachutes for emergency use must be of improved design if they are to withstand the severe pull encountered upon opening after ejection from the faster modern aircraft. Furthermore, the shock upon opening must not be so great as to injure the jumper. In addition to their usual function of enabling personnel to descend safely from disabled planes in emergencies, parachutes have now become an important tactical device for delivering men and equipment to territory not otherwise readily accessible. The development of safe parachutes to meet these various requirements calls for more complete knowledge of the strains produced in parachute shrouds and canopy and of the forces acting on the jumper or load. This parachute development work is greatly aided by equipment for making remote measurements on test-dropped dummy-carrying parachutes.

In the NBS parachute telemetering system, resistance strain gages are arranged to sense both altitude (pressure) and the tension in various harness straps. An inductive-commutator arrangement excites one strain gage at a time, the output of the gages being combined to modulate a small battery-powered radio transmitter. Seven measurement channels and a calibration channel are provided. At the ground station, a spot on the face of a cathode-ray tube moves up and down with the amplitude of the modulating signal. This spot is recorded on continuously moving photographic film. The film record is then transcribed, using a semi-automatic film reader, to obtain plots of altitude versus time and harness tension versus time.

The airborne part of the system is designed for ruggedness, low cost, and medium accuracy. Ruggedness is necessary because the period of maximum shock, when the parachute is opening, is the period for which measurements are most important. Low cost is desirable because, with experimental parachutes, occasional losses of equipment are to be ex-

* Communicated by the Director.

pected. Medium accuracy—of the order of 10 per cent—is sufficient, since the phenomena being measured are not reproducible with high uniformity.

The airborne transmitter operates on 217 megacycles and has an output of about 0.75 watt and a range of about 2 to 10 miles. Seven tubes are required for the entire airborne unit. The battery pack, which provides 7.2 volts at 1.0 ampere and 190 volts at 80 milliamperes, is tailored for about 1 hour of service.

The tensiometer which senses forces in the parachute harness is an H-shaped steel structure to which resistance-wire strain gages are cemented. Gage resistance is 1500 ohms, and the maximum useful change with harness tension is about 7.5 ohms. The altimeter consists of an evacuated bellows which responds to changes in air pressure by deflecting a thin cantilever beam carrying resistance strain gages. As both the tensiometer and the altimeter are linear to better than 5 per cent, point-by-point calibration is not needed.

Modulation of the transmitter with the signal from one channel at a time is accomplished by means of a "coder" of novel design. The coder supplies a series of time-sequential pulses to eight resistance bridges—one for the altimeter, one for each of six tensiometers, and one fixed bridge for calibration. One hundred pulses per second are received at each bridge.

The bridge-unbalance signals are first amplified and then converted to amplitude modulation of a 15-kilocycle subcarrier. This subcarrier in turn frequency-modulates the 217 megacycle transmitter by means of a reactance-tube modulator. A flexible transmitting antenna projects from the top of the dummy that houses all of the electronic equipment.

The ground-station receiver gives an output that is identical to the amplitude-modulated 15-kilocycle subcarrier in the airborne unit. Demodulator and filter circuits convert this output signal to a waveform identical to the original bridge-unbalance signals. This signal is applied to a cathode-ray oscilloscope and photographed on moving 16-millimeter film. The output of the ground station is thus a continuous film record of the eight original signals.

To make this record useful, it must be converted to individual curves showing altitude and the various harness tensions against a common time scale. This otherwise time-consuming conversion is expedited by a specially designed film reader.[1] Using this device, the operator adjusts potentiometer-connected mechanical tips to coincide with the signal peaks of the projected film image. When the tips have been so adjusted for one scan of eight channels, the seven information channels are automatically recorded on a strip chart. The recorder chart is advanced automatically, and the entire procedure is then repeated until all information of interest has been transcribed.

[1] For a somewhat fuller description of the film reader, see "Film Reader for Telemetered Information," NBS Technical News Bulletin **36**, 50 (April 1952).

PRECISE DETERMINATION OF ORTHOPHOSPHATE

Until now the precise determination of phosphate in rocks, bones, teeth, fertilizer, and other materials has been a difficult and time-consuming process because both gravimetric and volumetric methods require the complete separation of calcium and other interfering elements. The more rapid colorimetric procedures, while adequate for most routine purposes, do not have the precision that is often required. Recently, Drs. Allen Gee and V. R. Deitz at the National Bureau of Standards developed a rapid one-step procedure for orthophosphate determination which combines the essential simplicity of a spectrophotometric method with the precision of careful gravimetric analysis. Their work was done in connection with a research program sponsored at NBS by the Sugar Research Foundation, Inc.

The proposed method uses the techniques of differential spectrophotometry, but some refinements have been introduced. The method is based on the yellow complex (molybdivanadophosphate) formed when phosphate reacts with a mixture of molybdic and vanadic acids. Solutions of the yellow complex are made up to high absorbancy (less than 1 per cent transmission) in the near ultraviolet, and their absorbancy is compared with that of solutions containing known amounts of phosphate. While the absorbancy of the yellow solutions changes measurably with standing, this source of error is eliminated by comparing solutions developed simultaneously.

The procedure employs the Beckman DU Spectrophotometer with an incandescent lamp, a wavelength setting at 390 millimicrons, and a slit opening of about 1 millimeter. The selection of this wavelength permits the use of the glass filter supplied with the instrument to take out the stray light of wavelengths above 400 $m\mu$, which must be completely eliminated.

In practice, aqueous solutions of ammonium molybdate and ammonium metavanadate, as well as a standard phosphate solution containing perchloric acid, are prepared in advance and kept on hand. When a phosphate determination is to be made, the unknown material, ordinarily dissolved in hydrochloric acid, is diluted with water. Aliquots of this solution are taken which contain amounts of phosphate varying over a small range—say, 4.5 to 5.5 milligrams of P_2O_5 per 100 milliliters of final volume. At the same time the standard phosphate solution is used to make up two or three standard solutions covering this range. A solution containing the appropriate amounts of ammonium molybdate, ammonium vanadate, and perchloric acid is then added quickly to each flask which holds the unknown or standard solution, and the remaining volume is made up with water. A 1-cm absorption cell is filled with the lightest-colored solution, and all the other solutions are successively placed in a second cell and compared spectrophotometrically with the first solution. The phosphate content of the unknown

solutions is interpolated from the absorbancies and known phosphate contents of the standards. A linear extrapolation for measurements not bracketed by the standards can be made, though with somewhat reduced precision.

Under optimum conditions phosphate concentrations can be compared by this method with a precision of 1 part in 3000. Results are thus comparable with the best gravimetric procedures. With proper dehydration of silica, NBS standard phosphate rocks can be analyzed to an accuracy of 1 part in 1000. Since few substances (notably silicate and arsenate) interfere appreciably with the absorbancy measurement, the procedure should have rather wide application. NBS experiments indicate the possibility of adapting the process to various types of analyses.

NOTE: For further details, see "Determination of Phosphate by Differential Spectrophotometry," by Allen Gee and Victor R. Deitz, *Anal. Chem.*, Vol. 25, p. 1320 (1953).

The Franklin Institute, a privately endowed institution which exists today only because of the generous benefactions of its supporters, is justly proud of the dividends in human benefits accrued from the investments made through the years by its friends. The accumulation of many bequests for general endowment and income to be used where it will do the most good, is the most important and pressing need of The Franklin Institute today.

Checks should be made payable to The Franklin Institute and sent to The Franklin Institute, 20th and The Parkway, Philadelphia 3, Pennsylvania.

Transfer of property should be made to The Franklin Institute.

A typical form is:

I hereby give and bequeath to The Franklin Institute of the State of Pennsylvania the sum of.....to be used as The Board of Managers of The Franklin Institute may think advisable for the benefit of said Institution. (For the principal and income thereof to be devoted to.....).

Additional information on gift and memorial opportunities may be obtained through the above address.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING
February 17, 1954

The Stated Monthly Meeting of The Franklin Institute was held on Wednesday, February 17, 1954, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. Approximately 290 persons attended.

The President asked for approval of the Minutes of the December Stated Meeting, which were printed in the January JOURNAL. There being no corrections or additions, they were approved as submitted. He stated that the Minutes of the January Stated Meeting would appear in the February JOURNAL and would be submitted for approval at the next Stated Meeting.

The Secretary reported the following elections to membership for the month of January:

Active	23
Associate	26
Student	11

and a total membership, including Company and Library privileges, of 6,843 as of January 31, 1954.

The Secretary then read the following report:

ELLIOT CRESSON FUND
Report of the Trustees to
The Franklin Institute
February 17, 1954

The deed of gift under which Elliot Cresson in 1848 gave $1000 to two trustees, the income of which was to furnish medals to be awarded by The Franklin Institute, provided that the trustees should present to the annual meeting of the Institute in January a report showing the principal of the fund and its income and expenses during the preceding year. It was impossible to have such a report ready for the annual meeting and it is accordingly being presented at this time.

Principal of the Fund (representing the original $1000 plus transfers thereto from Unexpended Income and profit from disposition of investments)		$4,304.73
Invested as follows:		
$4000 U. S. Treasury 2½% bonds, due 6/15/1962–67 at book value	$4,134.35	
Cash in Western Savings Fund Society	170.48	
	$4,304.73	
Income of the Fund for 1953		
Interest on bonds (less amortization)	126.29	
Interest on bank balance	3.10	129.39
Expenses		
Medals, cases and certificates for award of three medals		265.26
Expenses in excess of income for the year		(135.87)
Unexpended Income at January 1, 1953		2,281.36
Unexpended Income at December 31, 1953		2,145.49
Invested as follows:		
$2000 U. S. Treasury 2½% bonds, due 6/15/1962–67 at book value	2,067.47	
Cash in Fidelity-Philadelphia Trust Company	78.02	
	$2,145.49	

/s/ Henry B. Allen
HENRY B. ALLEN
/s/ John Frazer *Trustees*
JOHN FRAZER

The Secretary pointed out that the Elliot Cresson Medal is the oldest awarded by the Institute and has been awarded nearly every year since 1875.

The President introduced the speaker for the evening—Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute and Senior Advisor to the Institute's Laboratories for Research and Development. Dr. Swann's illustrated lecture—"Beauty, the Ideal—of the Arts, and the Sciences"—was enthusiastically received by the audience and will be printed in a future issue of the JOURNAL.

The meeting adjourned at 9:50 P.M. with a rising vote of thanks to Dr. Swann.

HENRY B. ALLEN
Secretary

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JANUARY 20, 1954

ACTIVE FAMILY

Linn P. Brown, Jr.
Bayard C. Dickinson
Martin William Offinger
Edwin H. Thomas

ACTIVE

H. G. Bauer
William H. Ellis
Harry Granoff
Robert B. Harrison
Harvey A. Neville
Paul K. Perilstein, M.D.
Eugene R. Raup
Martin Schnabl
Charles S. Schroeder
Arthur G. Werther
A. Frederic Wetherill

NECROLOGY

C. Mason Astley, M.D. '49
Mrs. Emily Q. A. Ellis '36
Howard George, Esq. '44
Mrs. William P. Gest '36
William M. Gilbert '42
Jan Litynski '52
John F. McNelis '47
Milton K. Meyers, M.D. '48
V. S. Mollenauer '47
P. J. Robinson '43
Edward D. Smith, Sr. '42
T. Barclay Whitson '43

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, February 10, 1954.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 10, 1954.

PROFESSOR W. S. PARDOE in the Chair.

The following report was presented for final action:

No. 3257: Zirconium Concentrated-Arc Lamp.

This report recommended the award of a John Price Wetherill Medal to William D. Buckingham, of Long Island, New York, "In consideration of his invention of the Zirconium Concentrated-Arc Lamp, a device capable of producing a high intensity, sharply defined, extremely small beam of light and with modulation characteristics heretofore unobtainable in practical devices."

JOHN FRAZER,
Secretary to Committee

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

MOON, PARRY AND DOMINA EBERLE SPENCER: The Coulomb Force and the Ampere Force.
STIPPES, M. AND R. E. BECKETT: Symmetrically Loaded Circular Plates.
ALBRECHT, H. O. AND C. E. MANDEVILLE: The Storage of Energy in Some Activated Alkali Halide Phosphors.
MOON, PARRY AND DOMINA EBERLE SPENCER: A New Electrodynamics.
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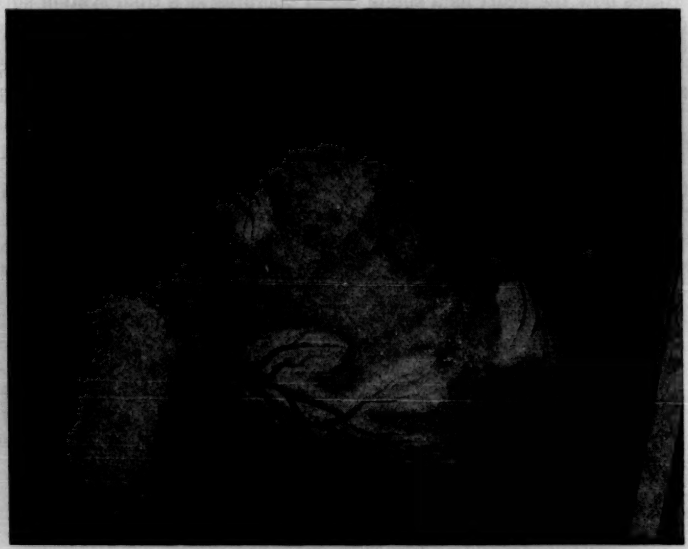
NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, EXECUTIVE DIRECTOR

THE HUMAN HEART

If a popular vote were to be taken on the nature of the exhibits our Museum visitors most desired to see presented for their instruction it is more than probable that the general judgment would lean heavily in favor of exhibits that would teach the visitor more about the construction of his own body and the function of its organs. Recollection of the increase in the attendance of visitors during the display of the Transparent Woman convinced the staff that this type of exhibit has a strong popular appeal. However, exhibits of this nature, it was realized, must be carefully chosen and scientifically presented in order that no mischievous or erroneous deductions be drawn from them by lay visitors.



Photograph by Photo Research

The Heart Exhibit

Note the head of the man peering over the top to obtain a conception of the size.

Accordingly, when the Department of Health of the Commonwealth of Pennsylvania approached the Institute with the suggestion that an exhibit on the human heart should be constructed, the proposal was warmly welcomed. The plans were ambitious, it being proposed to show more than any other exhibit of the kind had attempted. The projected giant heart involved working out a design employing techniques that were decidedly novel. Now that the plans have been realized visitors will agree that the original purpose has been amply fulfilled.

The construction is large enough for the visitor to walk about inside the heart. By following the prescribed path he will be following the course taken by a deoxygenated blood corpuscle coming from the lower part of the body and entering the right atrium. He will pass through the heart into the lungs where the corpuscle becomes oxygenated—changing in color—before passing back into the heart from which it is pumped through the arterial system as a completely oxygenated corpuscle.

The exhibit shows on a greatly enlarged scale the important relationship of the heart and lungs, and it furnishes a clear conception of the construction of the heart and the functions of its parts. It demonstrates how the heart as a single organ operates as a dual system supplying deoxygenated blood to the lungs and receiving the oxygenated blood back again for circulation through the body.

In order to make the picture more complete, two of the valves have been mechanized so that their operation may be better comprehended. The motion has been slowed down. To further enhance the realism, a sound effect is provided.

The exhibit has been generously sponsored by the Department of Health of the Commonwealth of Pennsylvania and the Heart Association of Southeastern Pennsylvania. It is now open to the public and should be included on the list of those things to be seen on your next visit to the Museum.

V.
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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

Seminar on The Separation of Biochemically Important Substances by Ion-Exchange Chromatography.—Dr. Waldo E. Cohn of the Biology Division, Oak Ridge National Laboratory, addressed the Seminar of the Biochemical Research Foundation on Tuesday afternoon, September 29, 1953.

Although ion exchange, as a chemical reaction, is over a century old and ion-exchange reactions (e.g., water softening) have been commonplace in chemical technology for decades, it has been only a few years since the first demonstration of ion-exchange chromatography of a precision capable of separating the closely related members of such families as the rare earths and the amino acids.

It is in the separation of each of the individual members of the conventional biochemical mixtures that ion exchange has made and promises to make its greatest contributions to biochemistry, contributions which far exceed in general utility the older applications to the isolation of single substances or of groups of related substances.

GENERAL CONSIDERATIONS

The most useful method for the separation of components in biochemical mixtures seems to be that of "elution analysis." This is a two-step process, wherein the mixture, previously prepared in a given form, is first adsorbed at the top of a column and then eluted in such a manner as to bring each substance to the bottom of the column as a separate and distinct "peak" without significant change in the form of the exchanger (in contradistinction to "displacement chromatography" where a change in the form of the column does take place).

The method of charge adjustment is obviously applicable only to ions with variable charge, a not unusual situation in biochemistry. Amino and carboxyl groups are readily ionized or deionized, phosphate esters become singly or doubly ionized, sugars form borate complexes, etc., at pH values that are not difficult to attain in the laboratory and usually are not destructive to the material under investigation.

AMINO ACIDS

For the separation of amino acids by elution analysis, either in one experiment or in consecutive experiments, four possibilities exist: a cation exchanger in either the acid or the salt form (with elution by acid or salts, respectively) or the use of an anion exchanger in either the base or the salt form (with elution by base or salts, respectively).

Although there is unquestionably an exchange of hydrogen or sodium ions, respectively, for amino acids in these two experiments, ionization of the acids is not the only factor entering into the distribution coefficient which determines the position of each component in the chromatogram. Glycine, alanine, valine, isoleucine, and leucine have the same pK_{B1} values (4.22, 4.13, 4.28, 4.24, and 4.25, respectively) and pI values (6.06, 6.11, 6.00, 6.04, and 6.04, respectively), yet they show a variable degree of separation in each instance even though the order is the same, which may indicate that the same complex of factors is operative in each.

NUCLEIC ACID DERIVATIVES

Nucleic acids may be degraded to their purine and pyrimidine bases, and the quantitative analyses of mixtures of these substances have occupied the attention of many investigators ever since their discovery. These substances, although classified as bases, are capable of functioning either as acids or bases although uracil and thymine have essentially no basic properties and cytosine and 5-methylcytosine have essentially no acidic properties. The other three heterocyclic compounds can be adsorbed and eluted from either anion exchangers or cation exchangers.

As anions, these five bases are separable by charge adjustment in a chloride (or formate) system. Here there are at least two aberrations from the order of the acidic dissociation constants. Adenine should appear with guanine; thymine (5-methyluracil) has a higher pK value than uracil and should precede it. In the latter case, the addition of the methyl group may explain the additional affinity which reverses the predicted order.

The ribosides and desoxyribosides of these bases partake of their behavior, with exception of adenosine, in which esterification of position 9 of adenine by ribose removes the most effective acid group of adenine and decreases markedly its adsorption by anion exchangers. Separations of uridine and cytidine by cation-exchange adsorption of the latter have been reported. A cation-exchange separation of four desoxynucleosides, utilizing ammonium acetate elution from a sulfonic acid cation exchanger in the ammonium form, has been utilized in the preparation of these substances; the volatility of ammonium acetate makes its removal very simple.

The degradation of nucleic acids to nucleotides, in which the above bases remain linked to ribose or desoxyribose which is in turn esterified with phosphate, results in a mixture of substances with properties which are such as to make separations by the classical techniques of precipitation very difficult.

The application of this technique for nucleotide analysis to the unpurified mixtures obtained from the hydrolysis of native nucleic acid led to the discovery of two isomeric forms of each nucleotide. One of these, the *b* form in each instance, was identical with the substances described earlier by Levene and his collaborators. In each case so far measured, it

is the more insoluble of the pair and has the lower pK value. Since it has been supposed that the known substances were 3′ phosphates, this led to the early belief that the new *a* substances were the 2′ isomers whose existence had long been made necessary by the hypothesis of a 2′–3′ internucleotide linkage. At the present time, however, it cannot be stated definitely which is the 2′ and which is the 3′, but the fact remains that it was ion-exchange analysis which led to the discovery of the isomeric compounds.

Enzymic degradation of ribonucleic acid or of desoxyribonucleic acid with diesterases has more recently been shown, again by ion-exchange analysis, to yield primarily the 5′ nucleotides and not the forms mentioned.

Doubly phosphorylated nucleotides, such as the pyrimidine diphosphates found in diesterase digests of ribonucleic acid and the adenosine polyphosphates involved in muscle contraction, are more strongly adsorbed to anion exchangers, as might be predicted from the increased number of acid groups, and require stronger reagents for their removal. Thus the pyrimidine diphosphates in the diesterase digests are easily separable from the monophosphates, and it has been shown that an easy and exact separation of the adenosine polyphosphates from each other and from muscle adenylic acid is possible.

Besides indicating the substances that comprise this digest, ion exchange can also produce these substances in quantities suitable for further study. Since at least two groups of tetranucleotides have been found, molecular weights up to 1200 seem to be amenable to conventional ion-exchange chromatography, with increased acid or salt to compensate for the increased number of acid groups.

SUGARS

Sugars are very weak electrolytes and have little tendency, in aqueous solution, to react with the salt form of ion-exchangers. Although it had been known for many years that certain polyhydroxyl compounds react with borate ions to form complexes which can ionize as acids, it was not until very recently that such complexes were considered for use in ion-exchange chromatography. These dissociation constants, in turn, are markedly influenced by the structures of the sugars involved. Thus, adjacent *cis*-hydroxyl groups form strong complexes, *cis*-hydroxyls once removed form weaker ones; *cis*-hydroxyls formed by the opening of a furanose or pyranose ring will be able to form complexes to the extent that the ring opens; and enolization of ketone groups will influence the number of reacting groups, as will mutarotation and furanose-pyranose interconversion, because of the greater degree of complexing of furanose structures.

Two examples of the power of this approach are shown. The separation of a mixture of pentoses and hexoses is shown. Although mixed

peaks are obtained, the isolation of each hexose from the other hexoses and of each pentose from the other pentoses is essentially complete.

The separation of a tetrasaccharide from a constituent trisaccharide, a constituent disaccharide, and a constituent monosaccharide, such as might be found in a partial hydrolysate of the parent molecule, has been achieved.

It is possible to recover each component as a pure sugar, free of borate, which makes the method suitable not only for analysis but for the preparation of pure sugars. It thus constitutes a new tool in sugar chemistry and biochemistry.

SUGAR PHOSPHATES, ORGANIC ACIDS, AND ALCOHOLS

The phosphorylated intermediates involved in glycolysis constitute a mixture which is difficult to resolve by classical methods. The discussion of the separation of the nucleotides makes it clear that diphosphates may be separated from those substances containing one or no phosphate groups, but the separation of the sugar monophosphates (fructose-6, glucose-1, glucose-6, and ribose-5) cannot be accomplished by chloride ions alone at any pH.

The sugar alcohols—sorbitol, dulcitol, and mannitol—are separable in exactly the same way as the neutral sugars.

Finally, the mixture of carboxylic acids encountered in the citric acid cycle may be considered. In the absence of phosphates and without the use of complexing agents, variation in ionic and organic properties and in the number of acid groups can be exploited for the separation of many of these substances.

PROTEINS

In the consideration of the ion-exchange chromatography of the amino acids, nucleotides, sugar derivatives, and carboxylic acids, it has been possible to indicate, with some degree of assurance, the roles played by ionic and nonionic forces with respect to the molecules in question. With proteins, lack of detailed knowledge of chemical structure precludes any such discussion and should, perhaps, exclude these substances from consideration in this review. Nevertheless, the practical importance of the column chromatography of proteins demands at least a mention of the achievements to date.

It has been shown that some protein solutions can be deionized, as in dialysis, by passage through ion-exchange beds without harm to the protein.

Several instances of the adsorption and elution of protein, undenatured and with hormonal or enzymic activities intact, have been reported. Among these are cytochrome c, ribonuclease, lysozyme, bacterial pectinase, and the adrenotropic hormone. In some of these cases the material has been readsorbed and reeluted without deviation from the original chromatogram, a further proof of the integrity of the desorbed molecule.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

FRICTION RESEARCH

BY

GEORGE ROBINSON

During the past several years, many of the projects at the Laboratories required information involving fundamental friction and lubrication phenomena. A separate section was started to aid not only other sections of the Laboratories but also outside interests in their lubrication

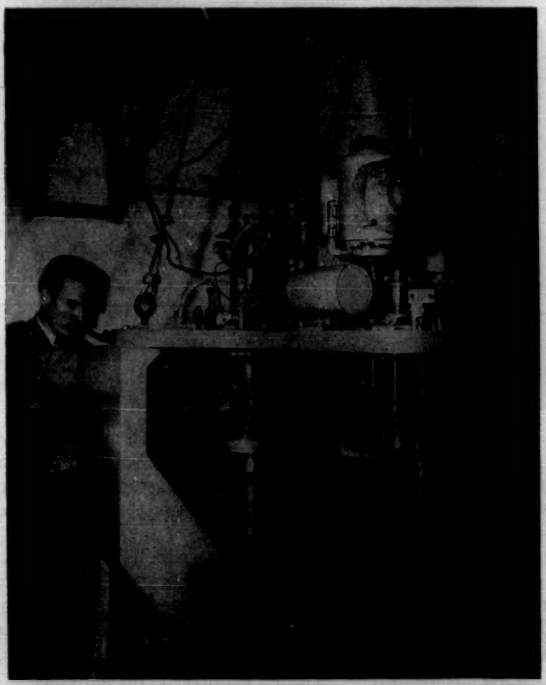

FIG. 1. The high-speed friction testing machine.

problems. In the short time that the Friction and Lubrication Section has been in existence, it has achieved valuable results in investigations in a number of fields.

In broadest terms, the scope of the section includes: fundamental studies of friction and wear under normal conditions as well as at extreme velocities and pressures; bearing materials research; design and development of bearings for special uses; and research on lubrication.

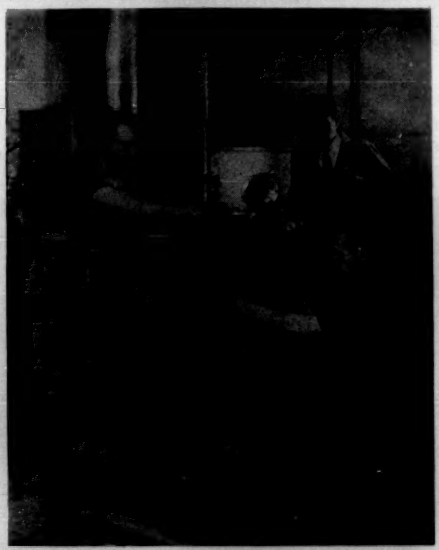

FIG. 2. Making adjustments on a machine designed by the Laboratories staff to test water-lubricated thrust bearings.

The first investigation undertaken was an intensive survey of the existing literature on dry sliding friction and boundary lubrication. This report contained the most complete bibliography of the literature on dry sliding friction ever published and has since its publication been kept up to date.

A special testing machine (see Fig. 1) which took three years to complete was designed and built at the Laboratories.[1] This machine has many unique characteristics and has extended friction and wear studies into hitherto unexplored areas. Whereas the greatest relative sliding speeds in friction tests have in general been limited to the neigh-

[1] A detailed description of this machine has been published in this JOURNAL, Vol. 256, August, 1953.

borhood of 18,000 feet per minute, this machine was designed for relative speeds as high as 120,000 feet per minute and has been successfully used for work at sliding speeds of about 100,000 feet per minute. The machine has another interesting characteristic in that bearing loads can be as high as 30,000 psi. This machine has already proved to be of great value in solving friction and wear problems in the field of interior ballistics and is expected to be of equal value in other friction applications where extreme load and speed conditions are encountered, or in exploring deeper into the fundamentals of sliding friction between solids.

FIG. 3. The AAR bearing test machine.

Another successful research project involved tilting-pad thrust bearings using water as a lubricant. Special test machines were designed and built in the Laboratories to impose prescribed conditions of temperature, load, and speed, and to record bearing performance (see Fig. 2). It was during the course of this investigation that the advantage of an unusual bearing shoe profile was discovered. This investigation gave valuable information regarding the selection of materials and the design of bearings which must operate in corrosive fluids of low viscosity.

Another important bearing project is currently under way. This project is aimed at determining the optimum bearing material for solid freight-car journal bearings (see Fig. 3). These bearings are subjected

to very severe conditions of operation on the railroads. In an effort to learn more about the fundamentals of their performance, the American Association of Railroads brought this problem to the Laboratories. Since this project is still in progress, however, results have not yet been published. We expect that the work on this project will eventually produce an evaluation of a wide range of bearing materials, with regard to their operation in the boundary and mixed-film lubrication region.

Professor Dudley D. Fuller of Columbia University has recently taken charge of the Friction and Lubrication Section. Before joining the staff of the Friction and Lubrication Section Professor Fuller was associated with the AAR project in the capacity of consultant. In addition to his work at Columbia, he has directed research projects for several major industries, and has published many papers in the field of Mechanical Engineering in the technical and trade press.

NOTES FROM THE BARTOL RESEARCH FOUNDATION

W. F. G. SWANN, DIRECTOR

ON H. ALFVÉN'S THEORY OF THE EFFECT OF MAGNETIC STORMS ON COSMIC RAY INTENSITY

BY

W. F. G. SWANN

For complete paper, see page 191 of this issue of the JOURNAL.

IONIC CONDUCTION IN THORIUM OXIDE *

BY

W. E. DANFORTH

One aspect of our cathode research program is concerned with mechanisms of disappearance of thorium oxide coatings in cathode service. One of the more prominent causes of such disappearance is electrolysis of the solid material by the current which must necessarily pass through it, hence the practical importance of understanding ionic conduction in this material.

When current is passed though a clear crystal of thorium oxide in vacuum at 1300° C., the interior becomes jet black and a metallic film forms on the surface. This film will eventually carry a large share of the current. In a practical cathode composed of a mass of small crystals the conduction is at first of semi-conductor nature.

Later, as a result of electrolytic formation of free metal, the conduction is in part metallic. The thermionic emission of the composite cannot be that of pure thorium oxide, but comes from a surface of complex nature in which the metal film plays an important part.

* Assisted by the Office of Naval Research and the Bureau of Ships.

NUCLEAR ENERGY LEVELS OF Tl^{203} *

BY

JAGDISH VARMA **

Using scintillation counting techniques, Pb^{203} is found to decay by K capture to the 683- and 280-kev excited levels in Tl^{203}. In addition to the 403- and 280-kev cascade gamma rays, the 683-kev cross-over transition is also observed. Careful energy measurements showed, in contrast with earlier results,[1] that the same 280 ± 1 kev level of Tl^{203} is excited in decay of both Hg^{203} and Pb^{203}. Conversion coefficient measurements of the 280-kev quantum support this conclusion. The K-shell conversion coefficient of the 403-kev gamma ray is measured as 0.052 showing the transition to be a mixture of 87 per cent E2 and 13 per cent M1. The K/L ratio also indicates the same admixture of E2 in M1.

The observed facts agree with orbital assignments of $d_{5/2}$, $d_{3/2}$, and $s_{1/2}$ for the levels of Tl^{203} in order of decreasing excitation energy, and of $f_{5/2}$ for the ground state of Pb^{203}. Angular correlation measurements give the anisotropy expected for a mixture-mixture correlation with mixing ratios as deduced from the conversion measurements.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

** Research Fellow, Bartol Research Foundation; permanent address, Morena (M.B.) India.

[1] G. D. O'Kelley, Ph.D. thesis, University of California. Unclassified Report URCL-1243 (May, 1951).

RESONANCE FLUORESCENCE WITH NUCLEI: Hg^{199} *

BY

F. R. METZGER AND W. B. TODD

The decay of Au^{199} proceeds partially through a 209 kev excited state of Hg^{199}. The 209 kev transition to the ground state of Hg^{199} has been used to excite Hg^{199} nuclei contained in a mercury absorber. The thermal motion in the Au^{199} source provided the velocities necessary to compensate through Doppler effect for the energy loss which the gamma rays suffer in the emission and absorption processes. When the source is heated, the contribution due to resonance fluorescence increases. From the observed change one calculates a lifetime of $(3.1 \pm 0.9) \times 10^{-10}$ sec for the 209 kev transition in Hg^{199}, assuming isotropic distribution for the M1 transition of unknown E2 admixture. The observed value has to be compared with the upper limit of 3.4×10^{-10} sec found with electronics[1] and with a value of 3.6×10^{-12} sec estimated from Weisskopf's lifetime formula.[2]

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] BELL, GRAHAM, AND PETCH, *Can. J. Phys.*, **30**, 35 (1952).

[2] WEISSKOPF, *Phys. Rev.*, **83**, 1073 (1951).

THE INNER BREMSSTRAHLUNG FOR Cs^{131} *

BY

BABULAL SARAF **

The continuous gamma-ray spectrum associated with orbital electron capture in Cs^{131} has been studied with the aid of a scintillation spectrometer. From the end point, the disintegration energy of Cs^{131} is estimated to be 300 ± 20 kev. Upper limits of 10^{-7} and 10^{-10} per disintegration are placed upon transitions leading to the excited levels of Xe^{131} of excitation energies 80 kev and 163 kev. Values of log ft calculated from the energy measurement and the estimates of intensities given above are in agreement with orbital assignments of $d_{5/2}$ and $d_{3/2}$ for the

respective ground states of Cs^{131} and Xe^{131}, $s_{1/2}$ for the excited level at 80 kev, and $h_{11/2}$ for the isomeric level of excitation energy 163 kev. The shape of the spectrum has been studied in detail and there is evidence that it is not in agreement with that predicted by the simple theory applicable to nuclei of low Z. The radiative capture probability of the decay process has been measured.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

** Research Fellow, Bartol Research Foundation; on leave of absence from Agra College, Agra, India.

THE LUMINESCENCE OF BERYLLIUM OXIDE *

BY

H. O. ALBRECHT AND C. E. MANDEVILLE

Under cathode-ray and heavy ion bombardment, beryllium oxide was described by Sir William Crookes as having "a beautiful blue phosphorescence." A preliminary survey of western publications[1] has shown that although BeO has been used frequently as a constituent in silicates and other phosphors, no detailed studies of the luminescent properties of "pure" unactivated BeO appear to have been described. It has recently been observed by the writers that when powdered BeO is irradiated by polonium alphas, fluorescence occurs in a broad emission band, extending from ~ 2000 Å to 4000 Å and above. A tremendous light yield is obtained in the deep ultraviolet ($2000 \text{ Å} < \lambda < 3000 \text{ Å}$), rivaling and sometimes exceeding that of silver-activated sodium chloride. A faint long period phosphorescence was excited by a Tesla discharge at an air pressure of ~ 10 microns. This effect was enhanced many fold by heating the pure BeO to 1400° C. in atmospheric air. The heat-treated material gives a very intense long-period ultraviolet phosphorescence. BeO is in general less luminescent under hydrogen-arc excitation than are the activated alkali halides.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] The literature of the Soviet Union and associated nations has not been thoroughly searched.

ANGULAR DISTRIBUTION OF NEUTRONS SCATTERED FROM CADMIUM, TIN, AND BISMUTH *

BY

S. C. SNOWDON AND W. D. WHITEHEAD **

The differential cross-sections for the scattering of 3.7 Mev neutrons from cadmium, tin, and bismuth have been measured over an angular range between 13 degrees and 140 degrees with an angular resolution of about ± 10 degrees. The effects due to higher order scattering have been removed at each measured angle by extrapolation of the observed

differential cross-section to zero thickness scatterer. A possible explanation of the results in light of the continuum theory of nuclear reactions will be given.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

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ENERGY DISTRIBUTION OF SECONDARY ELECTRONS PRODUCED BY 1.3 MEV PRIMARY ELECTRONS *

BY

R. A. SHATAS, J. F. MARSHALL AND M. A. POMERANTZ

Utilizing the linear accelerator as a source of 1.3 Mev electrons, the secondary electron emission from thin targets which do not stop the primaries has been investigated. In the usual thick-target experiments, scattered primaries and high energy secondaries are indistinguishable, whereas the former do not register in the present arrangement. Under these conditions, it has been possible to determine that energetic secondaries (delta-rays) constitute about half of the total yield. With a 0.002-in. Ni target the total yield from both faces (I_{sec}/I_{prim}) is 3.6 per cent, and the delta-ray yield exceeds 1.5 per cent. The latter is at least qualitatively in accord with theoretical expectation. The customary procedure of designating as "true secondaries" all electrons which fail to reach a collector when some arbitrarily-assigned retarding field is applied thus introduces appreciable errors; similarly it is erroneous to regard all energetic electrons leaving a bombarded target as scattered primaries. Preliminary measurements have also indicated that the energy-distribution in the low energy region is roughly similar to that observed when the bombarding energy is low, with the maximum for Ni occurring below 6 ev.

* Assisted by the Office of Naval Research and by the Office of Ordnance Research.

FORWARD-BACKWARD RATIO OF SECONDARY ELECTRONS EMITTED FROM THIN TARGETS *

BY

M. A. POMERANTZ, R. A. SHATAS AND J. F. MARSHALL

Preliminary measurements of the yield of secondary electrons emerging from the entrance and exit faces of thin metallic targets have been obtained. In the case of a 0.0005-in. Ni foil, the yields from both surfaces are equal, within an experimental uncertainty of approximately 20 per cent. It is hoped that the precision will be increased appreciably through the use of a sealed-off experimental tube, which can be processed in accordance with established high vacuum procedures for insuring the cleanliness of the target surface.

* Assisted by the Office of Naval Research and by the Office of Ordnance Research.

BOOK REVIEWS

DISLOCATIONS IN CRYSTALS, by W. T. Read, Jr. 228 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., 1953. Price, $5.00.

The recent appearance of two books on dislocations* graphically illustrates the rapid growth of this field of crystal imperfections. In fact, *Dislocations in Crystals*, by W. T. Read, Jr., requires 216 pages of very concise writing to present a "systematic development of what is *definitely known* of the subject."

This book is a triumph of simple physical explanations of the fundamental concepts of the dislocation theory. Although there is virtually no mathematics in the text, the reader is treated to a remarkably complete and understandable picture of the physics involved in the phases of the subject treated. One has the impression that if a treatise similar to this one were written as an introduction to more of the fields of solid state physics, we would not now have the severe need for more sound training in fundamentals on the part of many graduates.

The division of the (14) chapters into sections, which in general cover different subjects or alternate treatments of the same subject, is not allowed to interfere with the systematic development of the main topic of the chapter. The references, while not exhaustive, comprise all those works which introduced, or presented the accepted development of, the important phases of the theory. In general, the technical aspects of the book are good: it is adequately indexed, copiously and well illustrated, and printed in large type on good quality paper. The copy this reviewer received apparently had been treated rather unkindly by the binding machine, for several pages were marred annoyingly by tears.

An unusual feature for a book of this kind is the section of well chosen problems which appear at the end of each chapter, as well as the many suggested exercises in the body of the text. Some of these problems and exercises remind the reader to review fundamentals which cannot be covered properly in the text, while others provide needed extensions of material covered. In short, they provide the basis for a working knowledge of the subject which, in some cases, cannot be gained from the text due to the avoidance of a mathematical treatment.

It has been argued, and rightly so, that a scientific writer need not be a Shakespeare (or even a Richtmyer). However, it is this reviewer's opinion that if such obviously loving care were to be given to presentation of the fundamental physics of a given situation, comparatively little extra time could have considerably improved the literary merits of this book. There is no quarrel here with the punctuation or syntax—it is simply with the style which at times is very uneven. In what was apparently an attempt at conciseness, an admirable quality, the treatment occasionally becomes laconic.

Over all, however, the reviewer found this to be one of the most admirable introductory books he has encountered on any subject, and most of both the text and the problems will be required of his students.

F. E. JAUMOT, JR.

HIGH ALTITUDE RESEARCH, by Homer E. Newell, Jr. 298 pages, diagrams, 16 × 24 cm. New York, Academic Press, Inc., 1953. Price, $7.50.

Very early in the rocket firings, scientists realized that here was an unparalleled opportunity to explore the upper atmosphere. Very early came a wedding of the practical and theoretical sciences to yield the necessary research tools for this probe. From every part of this country and from some abroad scientists joined in this significant step to understand the mechanism of the atmosphere. The knowledge gained from this work is difficult to assess at this time—only the future will permit this period to be viewed in balanced perspective.

* The other is A. H. Cotrell, "Dislocations and Plastic Low in Crystals," Oxford, New York.

For those who have followed the literature on rocket soundings of the atmosphere there are found measures of temperatures, pressure, density, ionization and other characteristics of the atmosphere as we move away from the earth. In the back of the mind of the reader there arise the questions: "How do they know?", "What is the probable error of the observation?", "Are they off by a few percent or a few hundred percent?"

The detailed story of the probe of the atmosphere is carefully outlined and delineated in *High Altitude Rocket Research* written by Dr. Homer E. Newell, Jr., head of the Rocket-Sonde Research Branch of the Naval Research Laboratory.

Rocket probes of the atmosphere are relatively new. In April 1946 the first V-2 was blasted into the sky to telemeter back cosmic ray data. Since that date about 170 Aerobees, V-2's and Vikings have been used for this exploration. For these 170 rockets an entirely new complex of instruments had to be designed, engineered and tested to bring back fundamental data so that an atmosphere "model" could be calculated.

Dr. Newell has divided his book into 10 chapters of which the first is simply an introduction to the subject and the last deals quite hurriedly with photography from the high altitude rockets. The meat and substance are contained in the other eight chapters.

Chapter II describes the rocket as a research vehicle. It was early realized that in order to obtain usable data from the rocket its many motions in flight had to be analyzed. This, as the author shows, was a task of extreme importance and of no small magnitude. The chapter ends with the schedule of the firing of a Viking.

Chapter III deals with the recovery of data. The coding and transmitting—the receiving and decoding and the NRL and APL telemetering systems are described. These are the most extensively used of about 100 telemetering systems proposed for this intelligence transmission.

Chapters IV and V deal with the determination of pressures, densities and temperatures in the upper atmosphere. This reviewer was always intrigued by the methods scientists used to determine air pressures in a vehicle traveling at almost a mile a second. Here they are described and the method analysed. Temperatures, Dr. Newell indicates, are determined by ejecting grenades from a flying rocket and exploding them. Measuring the time of arrival of the blast at the earth's surface at five listening stations—arrayed 1000 feet apart—theoretically yields the temperature. The effect of atmospheric winds of high velocity on temperature determinations is real and presents a challenge to the scientist to analyze the interplay of diverse forces.

Radiations, both solar and cosmic, the composition of the high atmosphere and the ionosphere and the earth's magnetic field complete the book.

High Altitude Rocket Research is an authoritative, readable, highly informative book which will appeal to both the layman and the specialist in the rocket field. It is a stimulating book, skillfully assembled with a complete bibliography presented at the conclusion of each chapter. The complete mastery of the subject by the author makes it an indispensable reference book for the library.

I. M. LEVITT

THERMIONIC VALVES, by A. H. W. Beck. 570 pages, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1954. Price, $12.00.

During the war years when hundreds of physicists and college professors gathered at MIT's Radiation Laboratory to commence their arduous task, the development of radar devices and systems, a detailed knowledge of electron tube operation was their first need. To these men Beck's new book, *Thermionic Valves,* would have been a perfect gift. However, such good fortune could not have been theirs, for nearly half of the material contained in *Thermionic Valves* was to be discovered by these same researchers in the realm of high frequencies.

Bountiful answers to the problem of generating and amplifying ultra-high and superhigh frequencies were discovered during the 1940's. The reflex klystron, the cavity magnetron, the high-frequency triode, the traveling wave and beam interaction tubes, as well as television picture converters and storage tubes, made the decade of the forties stand out as important years in vacuum tube evolution. Beck's new book is then a collection

of factual data, theory and design calculations covering these late developments.

The favorite volume of those who took up the study of electron tubes and circuits a decade ago was Reich's "Theory and Application of Electron Tubes." Since then there have been only a few books devoted specifically to electron tubes: primarily Appleton, Eastman, and Spangenberg.

Several volumes of the MIT Radiation Laboratory Series specialized in microwave tubes to a greater extent than the sections of Beck's writing devoted to this subject. Hence, they represent a more extensive reference source on magnetrons, klystrons and microwave triodes. The most complete reference on traveling wave tubes is by J. R. Pierce. Thus it should be recognized that *Thermionic Valves* is not the best answer for the specialist wishing the most comprehensive treatment of any one tube class. However, it is a very good text.

Beck has apparently tried to include enough of the basic principles of emission, field theory, electron optics, transit time effects, and noise phenomena to make a foundation for an understanding of all types of tubes. Then, for completeness he has brought in briefly the usual receiving tube characteristics which Reich treats in more detail. Finally, the last two hundred pages are devoted to the new, high-frequency tubes in what is probably the best modern compilation.

This book demonstrates an interesting evolutionary step, for it has all but crowded out the conventional tubes, which are already obsolete in the eyes of the transistor-minded. A permanent position has been reserved for basic principles in the front of the volume, since for these there will be no obsolescence. Lastly, the book has moved on down the spectrum to where communication bandwidth is more plentiful and where the newer tubes have made their debut. It has therefore kept up with the times in every way and in so doing has justified the writer's effort.

C. W. HARGENS

DESIGN IN STRUCTURAL STEEL, by John E. Lothers. 454 pages, diagrams, 16 × 23 cm. New York, Prentice-Hall, Inc., 1953. Price, $10.00.

Except for steel company handbooks, books concerned solely with design in rolled structural steel sections are rather uncommon. With the proper background, many structures can be designed using only the steel handbooks. But without the experience, it may be difficult. *Design in Structural Steel* provides a very comprehensive background for all types of design in structural steel.

Professor Lothers of Oklahoma Institute of Technology has obviously spent a lifetime in structural steel design, for his book is unusually comprehensive. He emphasizes the practical rather than the theoretical, and uses more than 200 problems to illustrate various aspects of design. While the reviewer is not too familiar with all the books on structural steel design, it is doubtful if any book contains such a wide and at the same time thorough discussion of all the details of structural design. And there are some items, notably the method of analyzing eccentric brackets, which are in book form for the first time.

Beginning with rolled steel beams and girders, Prof. Lothers discusses the design procedure for all the details of structural design. Such rolled steel members as rods, bars, lacing and stay plates; such connection details as riveted and welded joints, gusset plates, lug angles and pin plates; such eccentric connections as riveted and welded brackets, beam-girder connections, knee-brace connections, and notched-beam connections are all described. Methods of designing eccentrically loaded base plates, built-up beams and plate girders, roof trusses and industrial-building bents, and steel bridges complete the subject matter. Throughout the book, the various building codes and steel specifications are frequently quoted and referenced so that one becomes acquainted with standard and current construction practice.

E. W. HAMMER, JR.

MATHEMATICAL METHODS FOR SCIENTISTS AND ENGINEERS, by Lloyd P. Smith. 453 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1953. Price, $10.00.

This textbook is intended primarily for the senior or first year graduate student in Physics. The author is Chairman of the Department of Physics at Cornell University. The subject matter of this book has been used as the basis of a one-year course in Mathematical Methods for physics majors.

The first five chapters comprise a condensed review of those concepts and methods usually covered in a course in advanced calculus, including the integral theorems of Green and Stokes.

In a space of four chapters there is an excellent treatment of the theory of functions of a complex variable. The elementary topics include power series, uniform convergence, Laurent series, Cauchy-Riemann equations, the Cauchy Integral Theorem and the Cauchy Integral Formula. The important concept of analytic continuation is treated briefly. An entire chapter is devoted to residues and complex integration. The theory of residues is applied to the evaluation of important types of definite integrals, including the case in which the integrand function has branch points.

A chapter is devoted to the methods of representation of functions by infinite series of functions. The Mittag-Leffler theorem for the expansion in series of rational fractions for functions with an infinite number of isolated poles is given and applied, as well as infinite product representations of certain functions. A chapter on applications of complex variable to potential and conformal mapping problems includes: methods of constructing the complex potential function, the Poisson Integral formula, Schwarz-Christoffel transformation, and the two-dimensional flow of an ideal fluid.

Linear algebra, matrices, linear transformations, linear vector spaces, and the transformation of quadratic, bilinear and Hermitian forms are covered in a single chapter. Transformations to principal axes are given special attention. The important formulae and methods of vector analysis and tensor analysis are treated briefly.

Orthonormal function systems are introduced in a general manner, and then illustrated by several examples. General methods are presented for approximating a function in the mean by a linear aggregate of orthonormal functions. A general procedure, analogous to the Schmidt orthogonalization process for constructing a basis for a vector space, is developed for constructing an orthonormal function set from an arbitrary set of linearly independent functions. The methods are applied to develop the various orthonormal function sets based on the Legendre polynomials, Hermitian polynomials, Laguerre and associated Laguerre polynomials, Bessel and Hankel functions. For each particular function the generating function is first expressed as a contour integral using the Cauchy integral formula; the contour integral representation of the particular function is then obtained via appropriate differentiation operations. A brief chapter is devoted to orthonormal function systems with a continuous spectrum, Fourier transforms, and applications.

Linear integral equations are surveyed in a thorough fashion. The classical methods of solution are outlined, including the power series method, the Fredholm method, and the Hilbert-Schmidt method. The method of deriving the integral equation associated with a boundary value problem is illustrated. Concluding chapters cover the methods of the calculus of variations and the methods of probability theory of particular use to the physicist.

Some errors appear in Chapter 10. Problem 142.1 reads "Reduce to normal coordinates the pair of forms

$$2T = m(x_1^2 + x_2^2 + x_3^2)$$
$$2V = \mu(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3).$$"

Here the author undoubtedly intends T and V to represent the kinetic and potential energy, respectively. The kinetic energy is certainly a function of the generalized velocities $\dot{x}_j$, requiring dots over the x_j in the expression for $2T$. The same error is committed in problem 142.2. A similar error appears in equations 141.1 and 141.2, which are valid expressions for the potential energy, but do not represent kinetic energy without dots over the vectors $\vec{x}$ and x.

The treatment of Vector and Tensor analysis is far too brief, considering the importance of these subjects in expressing physical laws and problems. Fortunately, these subjects are fairly well covered in texts on theoretical physics.

In spite of these criticisms, the author has done an excellent job of surveying several mathematical disciplines of central interest to the physicist and engineer. The topics of complex variable theory, orthonormal function systems, and integral equations are particularly well presented and might well serve as the basis for short courses in these sub-

jects. The book should prove extremely useful to physicists and engineers engaged in the mathematical aspects of physical problems. HOWARD H. BROWN

AN INTRODUCTION TO RELAXATION METHODS, by F. S. Shaw. 396 pages, 14 × 21 cm. New York, Dover Publications, Inc., 1953. Price, $5.50.

Relaxation methods have been called "a mathematics for the engineer" because they provide sufficiently accurate but not exact solutions to numerical problems of great complexity. In this introductory text, however, the author has chosen to discuss the solution of basic mathematical problems with only passing reference to their physical applications. While the merits of such a plan may be debated, it does make clear the scope and simplicity of these techniques, which might otherwise be obscured by discussions of problem formulation.

The breadth of application which relaxation methods have found is illustrated by the topics discussed: simultaneous algebraic equations, ordinary and partial linear differential equations, eigenvalue problems, integral equations and free surface problems. Considerable space is devoted to the equations of Poisson and Laplace as applied to plane potential problems, but it is unfortunate that discussion of the "quasi-plane-potential" equation has been omitted. The use of line, block and irregular star operators is clearly developed and illustrated by application to axisymmetric and multiply-connected regions. Tables of irregular star coefficients and a valuable bibliography are also included. Each topic is treated in a detailed manner with lavish use of diagrams and thoroughly practical suggestions for the computer. The typography and makeup of this volume are not commendable but will not diminish its considerable value to students of this subject. A. E. GIMSON

THE PHYSICS OF THE STRATOSPHERE, by R. M. Goody. 187 pages, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1954. Price, $5.00.

Physics of the Stratosphere, written by Dr. R. M. Goody, a fellow of St. John's College, Cambridge, is another of the Cambridge Monographs of Physics which have proved so interesting and profitable in the past. In this volume the author has tackled one of the more difficult aspects of present-day physics—the atmosphere.

In the preface the author clearly outlines the problem and the inherent difficulties for he writes. . . . "Controlled conditions are essential for a successful experiment, while with the atmosphere it is only possible to observe and to hope that the conditions relevant to a unique interpretation are either known or can be guessed. . . . While the laboratory worker would wisely reject (incompatible) results as untrustworthy, the atmospheric physicist must often accept them as the only results he is likely to obtain."

Throughout the remainder of the book the author shows how discouragingly true is his preface. For example, on page 34 Dr. Goody speaks of a semi-controlled experiment to determine temperatures in the atmosphere. When the experiment is completed the scientist has to assume that for several days the atmosphere had the same wind and temperature structure in order to derive the results. No reason is advanced why this assumption is taken but it is known that the entire structure of the temperature determination is valid only if this assumption is true. The author admits the results "are subject to fairly large errors."

The entire problem of atmospheric temperatures is one of the most challenging and complex faced by the scientist. There are a variety of parameters which directly influence the results obtained from any experiment designed to yield the general vertical structure of temperature. Some mentioned are wind, seasonal influences, composition, density, latitude variation and no doubt others exist which also exact an influence in the final result.

The author has covered this aspect of the problem with an authority which can arise only from long experience and complete familiarity with the subject.

In the other chapters of the book are covered the composition of the atmosphere, the ozone content and the methods employed in deriving the ozone distribution, the problem of winds and turbulence in the atmosphere and finally there is a section on the interaction of the atmosphere and radiation.

The book is written primarily for the physicist and is comprehensive in its treatment of the various facets of the atmosphere problem with especial emphasis on experimental methods and results. Because it is written for a higher level the book assumes the reader to be versed in both the terminology and mathematics of the subject.

A good bibliography at the back of the book includes all the references quoted in the text. This is interesting in that the reference is given by name and date instead of by number as is the custom in this country.

The book will be welcomed by rocket and guided-missile research scientists, by aeronautical research teams involved in high altitude testing, by meteorologists who are trying to decipher one of the most complex and important problems of our time—the weather.

I. M. LEVITT

PROTECTIVE ATMOSPHERES, by A. G. Hotchkiss and H. M. Webber. 341 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $7.00.

In many cases, modern furnace practice requires a protective atmosphere. This book is a practical manual designed to aid in the choice and use of the correct atmosphere for a given job. Various furnace conditions are discussed, such as prevention and reduction of oxides, carburization, decarburization and nitriding. Remedies for protective atmosphere ailments are outlined.

Although the book is written primarily for the manufacturing industry, it contains information of use to the research engineer who has occasion to utilize protective atmospheres in his furnace operations.

R. L. SMITH

CURRENT TOPICS

Electronic Translation.—Russian was translated into English by an electronic "brain" recently for the first time, in a demonstration at IBM World Headquarters in New York.

Brief statements about politics, law, mathematics, chemistry, metallurgy, communications and military affairs were submitted in Russian by linguists of the Georgetown University Institute of Languages and Linguistics to the famous 701 computer of the International Business Machines Corporation. And the giant computer, within a few seconds, turned the sentences into easily readable English.

A girl who didn't understand a word of the language of the Soviets punched out the Russian messages on IBM cards. The "brain" dashed off its English translations on an automatic printer at the breakneck speed of two and a half lines per second.

More than sixty Russian sentences were given to the "brain" altogether. All were translated smoothly in a demonstration performed jointly by Georgetown and IBM as a phase of IBM's endowed research in computation.

A handful of men had been individually engaged in research at various institutions for almost a decade to make a machine convert the meaning of words clearly from one language to another. No practical results were achieved until Georgetown a year and a half ago enlisted the aid of the most versatile electronic "brain" extant, the IBM 701.

This amazing instrument was interrupted in its 16-hour-a-day schedule of solving problems in nuclear physics, rocket trajectories, weather forecasting and other mathematical wizardry. Its attention was turned at brief intervals from these lightninglike numerical calculations to the altogether different consideration of logic in an entirely new and strange realm for giant electronic data processing machines: the study of human behavior—specifically, the human use of words. The result, as publicly proved, was an unqualified success.

"The potential value of this experiment for the national interest in defense or in peace is readily seen," Prof. Leon Dostert, Georgetown language scholar who originated the practical approach to the idea of electronic translation, declared to the group of scientists and United States government officials who witnessed the demonstration.

"Those in charge of this experiment now consider it to be definitely established that meaning conversion through electronic language translation is feasible."

Although he emphasized that it is not yet possible "to insert a Russian book at one end and come out with an English book at the other," Doctor Dostert predicted that "five, perhaps three years hence, interlingual meaning conversion by electronic process in important functional areas of several languages may well be an accomplished fact.

"Another obstacle to inter-cultural communication will then have been removed—another step taken toward greater comprehension," he noted. "For it is through the print of language that man has ever sought to communicate more widely with his contemporaries, more completely with posterity.

Multi-lingualism has, in part, hindered this quest. Electronic language translation is another stride forward in man's effort to reach his neighbors.

"Concretely, if electronic language translation makes possible, in due course, the translation into the languages of the less developed areas of the world, of the basic references and scientific literature in existence in Western languages, this in itself would be significant. The value to research of having current literature in scientific fields readily and promptly available in various idioms is another practical objective."

So far as IBM's big 701 was concerned, the demonstration could have been carried out with English and any one of a number of languages. Russian was chosen by the Georgetown linguist because present-day understanding of the Soviet by western countries is impeded by the relatively small number of students of Russian as opposed to a steadily growing accumulation of Russian textual material whose true significance cannot even be estimated until its content can be converted into English.

This overflowing reservoir of data about the Soviet Union is not the work of spies. It consists of openly published material available in or from Russia to any interested party: books and magazines, newspapers, technical journals.

The scientists interested in translating this material electronically have no warlike intent whatever. Their whole purpose is to improve communication. They chose scientific and technical subjects as their first source because that type of writing is done with words having highly specialized meanings, and it is possible to predict that if a word appears in a certain context the chances of its having a certain meaning are extremely high.

The same probabilities of accurate prediction occur in other fields of technical writing, such as medicine and engineering. Consequently, Doctor Dostert assumes that electronic translation will begin with separate dictionaries for each technical area, and that as experience with them grows, enough will be learned to permit accurate translation of our common everyday language, in which are such illogical and unpredictable words as "charleyhorse."

"Charley" is a nickname for Charles. "Horse" is a type of quadruped. But "charleyhorse" does not mean a horse named Charley. It means a muscular contraction which may take place in the calf. And "calf" in this context does not mean the offspring of a cow.

What the electronic translators have actually done is to create an entirely new electronic language. They have taken normal words and attached to them tags or signs which give each word a precision it does not usually possess. These signs actually denote rules of grammar and meaning. Although only six rules were used in the demonstration, the six were enough to cover all the words in all the sentences the 701 was asked to translate.

The IBM "brain" could translate only because these rule-tags were hitched onto normal words. For the "brain" cannot think independently. It can only perform tasks in obedience to detailed instructions prepared by human minds. And the minds of the Georgetown linguists (Dr. Dostert was assisted by Dr. Paul Garvin, a member of his Institute staff, just as the enormous detail work at IBM was done by Mathematician Peter Sheridan, under the supervision of Dr. Cuthbert Hurd, Director of IBM's Applied Science Division) could not give the "brain" dependable instructions until they themselves had worked out foolproof means

of telling in advance how to translate a word which had more than one meaning.

The six rule-tags were the solution. Those particular six were chosen because they have a broader effect on language translation than any other rules studied by the Georgetown linguists. Doctor Dostert estimates that it may take as many as one hundred rule-tags to translate scientific and technical literature in general. But, no matter how large the number becomes the six will remain basic.

The six rules govern transposition of words where that is required in order to make sense, choice of meanings where a word has more than one interpretation, omission of words that are not required for a correct translation, and insertion of words that are required to make sense.

Here is an explanation of the mechanics involved in the operation of one rule which governs transposition of words where such inversion is required in order to make sense.

We begin with the Russian *gyeneral mayor*. These two words must be reversed to arrive at the proper translation in English: *major general*.

The switch is assured in advance by attaching the rule sign 21 to the Russian *gyeneral* in the bilingual glossary which is stored in the machine, and by attaching the rule-sign 110 to the Russian *mayor*.

The stored instructions, along with the glossary, say "whenever you read a rule-sign 110 in the glossary, go back and look for a rule-sign 21. If you find a 21, print the two words that follow it in reverse order."

So the instant the "brain" is given *gyeneral mayor* to translate, it looks in the glossary, finds the two words there, reads the rule-sign 110, goes back and finds rule-sign 21, and automatically acts accordingly—all in the twinkle of an eye.

One more example will suffice to make the procedure clear. For it, let us take the rule governing the choice of meanings of one Russian word through the word which precedes that word in the Russian sentence.

The Russian word *nauka* means *science* in English. The Russian word *o* can mean either *about* or *of*. The proper English translation of *nauka o* is *science of*, not *science about*. But how can the "brain" know that?

It knows because, in its Russian-English glossary, *nauka* has affixed to it the rule-sign 242 and *o* carries the rule-sign 141. And the instructions in the "brain's" memory say "whenever you read the rule-sign 141, go back and look for 241 or 242. If you find 241, select the first English translation and print both words in the order in which they appear in the Russian sentence. If you find 242, select the second English meaning." Consequently, when the computer is given *nauka o* to translate, it reads the 141, looks for and finds the 242, chooses the second meaning given for *o* which is *of* and prints correctly *science of*.

After the six rules were formulated as the foundation of electronic translation, the linguists tried them out on themselves. First they wrote out sentences in Russian. Then they wrote out instructions as to how the rule-signs could be placed in the Russian-English glossary to lead to the proper English translation. After that, they gave the Russian sentences and the instructions to government officials and others in Washington who knew nothing about Russian or electronic "brains." The officials followed the instructions and came up with the right translations.

The first step in preparing IBM's magical computer to repeat this human

performance of a mechanical task was to write electronically, in plus and minus charges on a magnetic drum surface, 250 Russian words and their equivalents in English. Wherever a Russian word had more than one meaning, each meaning was given a rule-sign. This set of electronic words then constituted the dictionary to which the "brain" could refer.

The second step in preparing the 701 to translate was to store the detailed instructions—exactly like those the people in Washington had followed, except that these were written in electrical charges on the faces of cathode ray tubes in the 701's electrostatic memory.

All that remained to be done after that was to give the computer the Russian words to translate. The "brain" responded at the rate of one full sentence every six or seven seconds.

The experimental demonstration can be rated only as a scientific sample, or, as Doctor Dostert neatly phrased it, "a Kitty Hawk of electronic translation." Nevertheless, the success of the project contains enormous implications for both linguistics and electronics.

Students of language are now for the first time justified in undertaking serious study of language from a mechanical point of view. They have a practical reason now for trying to find out how language actually functions.

From the viewpoint of the electronic "brain," the language translation also has tremendous significance. It has been learned, for instance, that the formulation of logic required to convert word meanings properly even in a small segment of two languages necessitates two and a half times as many instructions to the computer as are required to simulate the flight of a guided missile.

What IBM's astonishing 701 actually did, in executing the Russian-English translation, was to create within itself a working model of another "brain" specially designed to handle logic instead of mathematics. Thus, the "brain" has crowned its previous reputation for superlative versatility with an even more lofty laurel. And in so doing it has produced its own "brain" child.

Gum Turpentine May Become a Practical Source of Peroxides.—Crowning several years of research by chemists is the development at the Naval Stores Station, Department of Agriculture, Olustee, Florida, of a simple, rapid, economical process for the production of pinane hydroperoxide in good yield and good concentration from gum turpentine.

The development, which should be commercially feasible, will make pine gum, a plentiful crop, potential source of organic peroxides, suitable for use in a number of industrial processes, especially as polymerization catalysts in the production of synthetic rubber, resins, and plastics.

Terpene peroxides from gum turpentine produced by these scientists in earlier research proved to be very good polymerization catalysts in the production of "cold" rubber when tested by the Office of Rubber Reserve. But the practical method was needed to suit the development to commercial adoption.

Industry is interested. Several chemical companies have shown interest in producing pinane hydroperoxides and other turpentine peroxides. These could, in an emergency, replace cumene hydroperoxide, a benzene compound, commercial catalyst in "cold" rubber production.

G. S. Fisher, who helped develop the new, feasible process, describing it at the recent Regional Conclave of the American Chemical Society in New Orleans, Louisiana, said that 4 main

steps are involved: Hydrogenation of the turpentine to give pinane; purification of the pinane, usually by simple distillation; oxidation of the pinane with molecular oxygen to a peroxide content of about 50 per cent; and stripping the oxidate under vacuum to recover the unoxidized pinane, leaving the pinane hydroperoxide as a residue. He said that starting with 8-lb. batches of turpentine, this process yielded 4-lb. batches of peroxides about 80 to 90 per cent pure.

In the basic research through which the practical development was reached, chemists prepared a pure form of pinane hydroperoxide and established its chemical structure as the *cis-l*-pinane 2-hydroperoxide.

Results of Seven-Year Truck Research Program.—After seven years of testing in a truck research project, University of Wisconsin engineers report that in many ways, the all-wheel drive vehicle is superior to either rear-wheel or front-wheel drives.

Here are some of the points the research revealed:

1. In tractive ability—the ability to get a vehicle going—the all-wheel drive is superior, followed in order by the rear-wheel, then the front-wheel drive;

2. Fuel consumption of the all-wheel drive and the two-wheel drive, either front or rear, is essentially the same;

3. Over-all tire wear on both straight trucks and truck tractors ranges from 25 to 37 per cent less on all-wheel drives.

The UW truck research project, under the direction of Prof. Archie Easton of the University College of Engineering staff, has been supported by four Wisconsin industrial firms. During its seven years, the tests have been conducted throughout the year, with special winter driving tests each year on the ice and snow of Pine Lake near Clintonville, Wis.

The project has shown that "skidding" on wet, slippery pavements is almost eliminated with four-wheel drive, and "jackknifing"—literally sure death on the highway when the truck cab suddenly goes off at a sharp angle to the truck trailer—is almost impossible with a four-wheel drive tractor under power.

"Of the three types of drive, the rear-wheel drive is most unstable when the drive wheels are over-powered," Easton says. "This is characterized by 'fishtailing'; skidding, or loss of control on both a curve and on a straight-away. Spinning the drive wheels on a front or all-wheel drive causes no 'fishtailing' or skidding on the straight-away, and no tendency to go into a flat spin or skid on a curve.

"With a moving truck tractor on a slippery surface, the rear-wheel drive unit is limited to recovery from a maximum jackknife angle of about 35 degrees, which approximates the maximum steering angle," he says.

"With front or all-wheel drive, it is possible to recover from a jackknife angle approaching 90 degees on a slippery surface," he reports. "It follows that if recovery from such high angles is possible, the tendency to go into a skid or jackknife is at a minimum with front or all-wheel drive."

Easton's research has shown that steering is about equally easy and effective for all types of drive—front, rear, or all-wheel—but he says that power delivered through the steering wheels minimizes loss of steering control by preventing the locking of the front wheels during a braking operation.

He reports that the "fanning" technique of brake application, which is effective in reducing braking distance with rear-wheel drive, is even more

effective with all-wheel drive on a slippery surface, because there is power as well as brakes on all four wheels.

Considering the four-wheel vehicle, the four-wheel drive is to acceleration —speed and power—what four-wheel brakes are to deceleration—slowing down and stopping—he says.

Electrostatic Air Cleaner.—A new ceiling-suspended model of the home Precipitron® electrostatic air cleaner has been announced by the Westinghouse Electric Corporation. The average-sized home model, PH-122, is designed for simple installation in the main return air duct of a forced-air heating system.

The Precipitron is the Westinghouse-developed electronic device that removes 90 per cent of all airborne dust and pollen. The unit is effective even against tobacco smoke particles which are as small as 1/250,000th of an inch in diameter.

Weighing only 200 lb. (one-third less than its predecessor), model PH-122 fits the needs of the average five- to seven-room house. It handles from 1000 to 1200 cu. ft. of air per minute. For larger homes, a slightly larger model, PH-242, is available, which handles from 2000 to 2400 cu. ft. of air per minute. Both units clean recirculated air from the living quarters as well as "make-up" air from the outside.

The new home units may be operated for slightly less than it costs to burn a 60-watt light bulb. Maintenance is low, since there are no moving parts, and electronic tubes are guaranteed for one year.

Water and drain line connections for washing away accumulated dirt are provided on both units. The only electrical requirement is a single outlet on any alternating current, 115-volt line. Both Precipitron air cleaners may be adapted quickly to any forced-air heating system or residential air-conditioning plant. Compact and encased in a gray enamel finished steel cabinet, all major internal components are made of aluminum.

Stone Saw Adapted to Cut Glass Melamine Laminates.—A new and faster method of cutting glass melamine, made possible through special adaptation of a Stone Saw, has been introduced at the Boston Naval Shipyard. Besides cutting glass melamine up to two inches thick at the rate of four feet per minute, this versatile tool is reported to be easily adaptable for cutting granite, marble, tile, slate, brick, structural glass, plastic, refractories, and asbestos products. By attaching abrasive cutting wheels to the machine, many types of metal, wet or dry, can be cut with greater speed and accuracy than by conventional methods. The cutting wheel has proved very durable in more than a year of service.

New Devices to Disassemble Steam Control Valves on Turbines.—Several devices have been developed by the San Francisco Naval Shipyard for use in disassembling steam control valves (which regulate turbine rotor speed) on high pressure turbines. An initial problem of damage to the cap and disk of control valves has been overcome, and the holding devices have also been found useful for such operations as removing disk nuts, disk and stem assemblies of steam steps, and astern and ahead throttle valves.